Asymmetric Information, Portfolio Managers, and Home Bias*

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Abstract

We propose a model of delegated asset management that can explain the following empirical regularities observed in international markets: (i) the presence of home bias, (ii) the lower proportion of mutual funds investing domestically, and (iii) the higher ability and market value of mutual funds investing domestically. In the model, heterogeneous fund managers choose whether to specialize in domestic or foreign assets. Individual investors are uncertain about managers' abilities to generate abnormal returns, and they are more informed about domestic markets than foreign markets. As a result, they are better able to evaluate the ability of managers who specialize in domestic assets. This makes domestic investments less risky and generates home bias. Home bias is magnified by the managers' specializations: since ability is rewarded more in the domestic market, higher ability managers invest domestically, making domestic assets more attractive to the investors.

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1 Introduction

Nowadays a large share of international investments is executed by portfolio managers in financial institutions. By 2009, mutual funds, pension funds, and other financial intermediaries had discretionary control of about 75 percent of the US equity market.[1] However, most models of international finance rely on individual investors. We propose a partial-equilibrium model of delegated asset management that can explain the following empirical regularities in international markets: (i) the presence of home bias, (ii) fewer mutual funds investing domestically than internationally, and (iii) the higher ability and market value of mutual funds investing domestically.[2]

The model relies on two arguably reasonable assumptions, which are supported by empirical observations. First, individual investors have more precise information about domestic markets (see Ivkovich and Weisbenner, 2005; Malloy, 2005; Bae et al., 2008), while professional portfolio managers are equally informed about all markets. Second, there is unobserved heterogeneity in portfolio managers' ability to generate abnormal returns (see Chevalier and Ellison, 1999; Gottesman and Morey, 2006).[3]

More precisely, we study a two-period economy with two types of assets: domestic and foreign. In the first period, each manager chooses to specialize either in domestic or in foreign assets, and the decision as to which type of assets to specialize in is irreversible. Then, individual investors allocate their capital across managers based on imperfect signals about these managers' abilities. After the first period, they observe the excess returns generated by each manager, update their beliefs about the managers' abilities, and decide on the second-

[1] According to the Commission on Corporate Governance of the New York Stock Exchange (July 23, 2010, p. 12, http://www.nyse.com/pdfs/CCGReport.pdf).

[2] See French and Poterba (1991), Tesar and Werner (1995), and Ahearne et al. (2004) for evidence on home bias. Ahearne et al. (2004) find that the share of domestic equity in the US portfolio in the year 2000 is around 88 percent, while its share in the world portfolio is 50 percent. See Chan et al. (2005) and Hau and Rey (2008) for the remaining two regularities.

[3] The model is closely related to theories of delegation of portfolio management decisions such as Berk and Green (2004), Kothari and Warner (2001), Lynch and Musto (2003), Basak et al., (2007), Cuoco and Kaniel (2011), Mamaysky et al. (2007), Wei (2007), Dang et al. (2008), Garcia and Vanden (2009), and Glode (2011).

period allocation of capital. In what follows we describe the main findings of the paper.

Our model generates home bias. In the model, individuals invest via equally informed mutual funds and are equally informed about the funds' abilities to generate excess returns. However, investors update these beliefs using information on funds' excess returns. Since they know more about the domestic market, they identify with higher precision the abilities of funds that specialize in domestic assets. This in turn allows investors to allocate capital more efficiently among such funds and makes investments in domestic assets more attractive.

More interestingly, home bias is magnified by the reaction of managers to this initial asymmetry: i) since investors channel more capital to domestic assets, managers have higher incentives to specialize in those assets; ii) since investors can more precisely identify the abilities of funds specializing in domestic assets, better managers have the highest incentive to invest domestically. As a result, in equilibrium the most skilled managers specialize in domestic assets. Clearly, this incentivizes investors to channel even more capital to domestic assets.

The endogenous specialization by managers—in particular, the fact that managers with higher abilities specialize in domestic assets—allows us to generate two other findings consistent with empirical regularities.

First, managers specializing in domestic assets generate higher abnormal returns; as a result, they attract more capital. Hence, our model predicts that the market value of mutual funds specializing in domestic assets will be higher.

Second, our model predicts that there may be fewer mutual funds specializing in domestic assets than those specializing in foreign assets. If a manager with average ability invests domestically, she will compete directly with high-ability managers; hence, she is likely to receive only a small fraction of the capital going to domestic assets. If she invests in foreign assets, however, she will directly compete with low-ability managers, thereby receiving a large fraction of the capital going to foreign assets. Hence, average-ability managers prefer to specialize in foreign assets, and the equilibrium number of funds specializing in domestic assets decreases.

The idea that asymmetric information of individual investors about markets can explain the equity home bias puzzle is not new (see Gehrig, 1993; Brennan and Cao, 1997; Zhou, 1998; Barron and Ni, 2008; Hatchondo, 2008; and Van Nieuwerburgh and Veldkamp, 2009).[4] Because individuals watch domestic television, listen to domestic radio, and read domestic newspapers, they have more precise information about domestic assets' payoffs, and hence, investing domestically carries less risk when these individuals invest on their own. However, this explanation of home bias does not take into account the fact that most individual investments are executed by professional portfolio managers, and the latter are unlikely to face informational asymmetry: it is similarly costly to hire domestic and foreign experts or to perform detailed analyses about domestic and foreign markets. Indeed, we show that if portfolio managers are equally informed about all markets, then the asymmetric information of individual investors is not sufficient to generate home bias. It is the uncertainty about managers' abilities that provides a channel through which the asymmetric information faced by individual investors translates into asymmetric investment decisions. Our model thus restores the validity of the information-based explanation of home bias.

When the analytical results are ambiguous, we solve the model numerically and show that it can account for the three empirical regularities, mentioned above, about mutual the funds industry. For parameters for which the model predicts that 46 percent of funds invest domestically—an empirical observation made by Chan et al. (2005) and Hau and Rey (2008) – the model implies that individual investors channel 76 percent of their capital to domestic

[4] Following Sercu and Vanpee (2008), there are four more types of explanations for the equity home bias puzzle: the lack of perfect financial integration (see Black, 1974; Stulz, 1981; Martin and Rey, 2004); investors hedging domestic risks (see Cooper and Kaplanis, 1994; Baxter and Jermann, 1997; Heathcote and Perri, 2007; Coeurdacier and Gourinchas, 2011; Engel and Matsumoto, 2009; Coeurdacier et al., 2010); corporate governance, transparency, and political risk (see Dahlquist et al., 2003; Ahearne et al., 2004; Gelos and Wei, 2005; Kraay et al., 2005; Kho et al., 2009); and behavioral-based stories of equity home bias (see Barber and Odean, 2001 and 2002; Huberman, 2001; Karlsson and Norden, 2007; Solnik, 2008; Morse and Shive, 2011).

assets, and the ratio of the average market values of funds specializing in domestic assets and international funds is 2.2. The analysis also implies that managers investing domestically generate excess returns between 46 and 60 basis points higher than those investing in foreign assets. Although our model underestimates the empirical home bias, it nevertheless comes very close, given its stylized nature.

It is worth stressing that the local monitoring advantage of investors is directly responsible for only a small fraction of our quantitative results. We show that if managers were assigned exogenously to foreign and domestic assets, the asymmetry in how investors allocate their capital across markets would be small, and its direction would be ambiguous. Most of our results are driven by the managers' response to that asymmetry.

This paper suggests that both asymmetric information at the individual level and uncertainty about managers' abilities to generate abnormal returns can explain a range of empirical observations. Moreover, it provides a novel prediction that managers specializing in domestic assets are better at obtaining abnormal returns. Although we find evidence consistent with this prediction, we argue in the conclusions that the existing empirical literature is not well suited to test it. Hence, this paper leaves this empirical question open.

The paper is organized as follows. In Section 2, we present the model and discuss its assumptions. In Section 3, we analyze the equilibria for three versions of the model. In Section 3.1, we first analyze a benchmark version of the model in which we assume that there is no uncertainty about managers' abilities. The purpose of this exercise is to show that in the presence of mutual funds, asymmetric information about markets is not enough to generate home bias. In Section 3.2.1, we assume that managers' specialization is fixed at the specialization from the equilibrium in the benchmark model, but we reintroduce uncertainty about managers' abilities. We show that this creates home bias. In Section 3.2.2, we analyze the full model and show that the endogenous managers' specialization magnifies the initial home bias and may explain other features of the delegated portfolio-management industry. In

Section 4, we use numerical solutions to demonstrate the size of the effects identified in the model. Section 5 discusses the robustness of the results to our assumptions. In Section 6, we relate our results to the empirical literature and conclude. All proofs are in the appendix.

2 The Model

In this section, we first present the model and then discuss its most important assumptions.

We study a two-period economy with a large number Z of investors and a large number Z of managers. There are two types of assets in this economy: domestic (D) assets and foreign (F) assets.

At the beginning of the game, each manager either opens a mutual fund that specializes in domestic assets, opens a mutual fund that specializes in foreign assets, or does not open a mutual fund. There is a fixed cost F_M for specializing in assets M, where $M \in \{D, F\}$. Each manager specializes in the assets of one market only, and the decision about specialization is irreversible. Let $I_M(Z)$ denote the set of managers who open a fund specializing in assets M. Throughout the paper, managers specializing in domestic assets are called *domestic-asset managers* (or *domestic-asset mutual funds*) and constitute a domestic-asset market, and managers specializing in foreign assets are called *foreign-asset managers* (or *foreign-asset mutual funds*) and constitute a foreign-asset market. The reader should keep in mind, however, that all managers serve only domestic investors.

Each investor draws a fixed number T of mutual funds and does so uniformly and without replacement. The draws are independent across investors, and the set of mutual funds observed by each investor is constant for both periods. For each investor, let $N \in \{0, ..., T\}$ denote how many domestic-asset mutual funds she observes. Each period, each investor allocates a unit of capital between the observed mutual funds. At the end of each period, she consumes the obtained returns. Investors have mean-variance preferences over consumption with a coefficient of absolute risk aversion γ.

Managers maximize the amount of received capital minus the fixed cost of operating a fund F_M.[5] The payoff from not opening a mutual fund is normalized to 0.

In each period, each mutual fund j specializing in assets of market M obtains excess returns over a passive benchmark given by $R_{tj}^M = \alpha_j + \varepsilon_{tj} + v_{Mt}$. The first element α_j, called *ability,* represents the individual asset-picking skill that manager j brings to the table. The ability of each manager is independent of whether she specializes in domestic or foreign assets, is constant over time, and is independent of other managers' abilities. The second element ε_{tj} represents an idiosyncratic shock to excess returns of manager j. We assume that $\varepsilon_{tj} \sim N\left(0, \sigma_\varepsilon^2\right)$ is independent over time and across managers. The third element $v_{Mt} \sim N\left(\bar{\nu}, \sigma_\nu^2\right)$, called *fundamentals,* measures the excess returns that could be generated using active strategies based on publicly available information about assets M.

The following assumptions are crucial to our results.

Assumption A (asymmetric information) Investors observe domestic fundamentals, but do not observe foreign fundamentals.

Assumption B (uncertainty about managers' abilities) The ability of each manager consists of a publicly observed signal y_j and an unknown, random factor η_j; that is, $\alpha_j = y_j + \eta_j$, where $\eta_j \sim N\left(0, \sigma_\eta^2\right)$.

The game proceeds as follows. The first period is divided into four stages. First, managers observe the signals $\{y_j\}_{j=1}^Z$ about all managers' abilities. Then, they decide simultaneously whether and what kind of mutual funds to open. After that, each investor draws T mutual funds, observes y_j for each of her funds, and chooses how to allocate her first-period capital across these funds. Finally, the fundamentals and the excess returns are realized, and consumption takes place. In the second period, each investor observes the realized excess returns

[5] We are implicitly assuming that investors pay managers a fixed fee normalized to 1 per unit of capital invested and that managers do not discount future.

of the mutual funds with which she had invested, $\left\{R_{1j}^{D}\right\}_{j=1}^{N}$ and $\left\{R_{1j}^{F}\right\}_{j=1}^{T-N}$, and the first-period realization of domestic fundamentals v_{D1}.[6] Then investors update their beliefs about managers' abilities and allocate their second-period capital accordingly. After this, the fundamentals and the excess returns are realized, consumption takes place, and the game ends. In the next sections, we describe each stage of the game in more detail.

2.1 An investor's problem

Let $x_{jt}^{M}(N)$ be the amount of capital that an investor with N allocates in period t to manager j who specializes in assets M. Let $\phi_{jt}^{M}(N)$ be the belief that she holds in period t about the ability of this manager. Let $\sigma_{\alpha Mt}^{2}(N)$ be the variance of this ability conditional on the information available at time t. In what follows, we omit the argument N when confusion should not be an issue.

Each period, each investor solves

$$\begin{aligned}\max_{\left\{x_{it}^{D}\right\}_{i=1}^{N},\left\{x_{it}^{F}\right\}_{i=1}^{T-N}} & \sum_{i=1}^{N}\left(\bar{\nu}+\phi_{it}^{D}\right)x_{it}^{D}+\sum_{i=1}^{T-N}\left(\bar{\nu}+\phi_{it}^{F}\right)x_{it}^{F}+ \\ & -\frac{1}{2}\gamma\left(\left(\sum_{i=1}^{N}x_{it}^{D}\right)^{2}\sigma_{v}^{2}+\sum_{i=1}^{N}\left(x_{it}^{D}\right)^{2}\left(\sigma_{\alpha Dt}^{2}+\sigma_{\varepsilon}^{2}\right)\right)+ \\ & -\frac{1}{2}\gamma\left(\left(\sum_{i=1}^{T-N}x_{it}^{F}\right)^{2}\sigma_{v}^{2}+\sum_{i=1}^{T-N}\left(x_{it}^{F}\right)^{2}\left(\sigma_{\alpha Ft}^{2}+\sigma_{\varepsilon}^{2}\right)\right),\end{aligned} \tag{1}$$

subject to $\sum_{i=1}^{N}x_{it}^{D}+\sum_{i=1}^{T-N}x_{it}^{F}=1$.

2.2 A manager's problem

At the beginning of the game, each manager has a conjecture about the equilibrium specialization of managers. Based on this conjecture, she decides on her specialization. For any managers' specialization with the corresponding $I_D(Z)$ and $I_F(Z)$, let $\mu_Z \equiv \frac{|I_D(Z)+I_F(Z)|}{Z}$

[6] We abuse the notation slightly: when discussing a particular investor, we index the domestic-asset funds she observes by $j\in\{1,...,N\}$ and the foreign-asset funds she observes by $j\in\{1,...,T-N\}$, but these managers may be different for different investors.

denote the fraction of managers who open a mutual fund, and $n_Z \equiv \frac{|I_D(Z)|}{|I_D(Z)+I_F(Z)|}$ denote the fraction of mutual funds which invest in domestic assets. Let $\bar{y}_Z^M \equiv \frac{1}{|I_M(Z)|}\sum_{j\in I_M(Z)} y_j$ be the average expected ability of managers specializing in assets M.

Consider manager j who in the conjectured equilibrium specializes in assets D. If she encounters an investor that observes N domestic-asset managers, then she will receive from this investor x_{jt}^D in period t. Since each investor draws T funds uniformly and without replacement, the probability that a random investor draws j and $N-1$ other domestic-asset funds is $P_D\left(N|\mu_Z, n_Z, Z\right) = \frac{\binom{\mu_Z n_Z Z-1}{N-1}\binom{\mu_Z(1-n_Z)Z}{T-N}}{\binom{\mu_Z Z}{T}}$. Investors draw assets independently, so the expected number of investors who observe j and $N-1$ other domestic-asset funds is $ZP_D\left(N|\mu_Z, n_Z, Z\right)$. Hence, if manager j decides to specialize domestically, her expected profit is

$$\Pi_t^D\left(y_j; \mu_Z, n_Z, \bar{y}_Z^D, \bar{y}_Z^F, Z\right) = \sum_{N=0}^{T} ZP_D\left(N|\mu_Z, n_Z, Z\right) E\left[x_{jt}^D\left(N\right)|y_j, N, \bar{y}_Z^D, \bar{y}_Z^F, Z\right]. \quad (2)$$

If, instead, she decides to deviate to specializing in assets F, there will be more foreign-asset managers, and therefore the probability that she encounters an investor that observes N domestic-asset managers is $P_{F,d}\left(N|\mu_Z, n_Z, Z\right) = \frac{\binom{\mu_Z n_Z Z-1}{N}\binom{\mu_Z(1-n_Z)Z}{T-N-1}}{\binom{\mu_Z Z-1}{T}}$. Hence, if manager j deviates to assets F, her expected profit is

$$\Pi_t^{F,d}\left(y_j; \mu_Z, n_Z, \bar{y}_Z^D, \bar{y}_Z^F, Z\right) = \sum_{N=0}^{T} ZP_{F,d}\left(N|\mu_Z, n_Z, Z\right) E\left[x_{jt}^F\left(N\right)|y_j, N, \bar{y}_Z^D, \bar{y}_Z^F, Z\right]. \quad (3)$$

The formulas for $\Pi_t^F\left(y_j; \mu_Z, n_Z, \bar{y}_Z^D, \bar{y}_Z^F, Z\right)$ and $\Pi_t^{D,d}\left(y_j; \mu_Z, n_Z, \bar{y}_Z^D, \bar{y}_Z^F, Z\right)$ are derived analogously. Similarly, one can derive the amount of capital that manager $j \notin I_D\left(Z\right) \cup I_F\left(Z\right)$ expects to receive in t, if she deviates to opening a mutual fund that specializes in assets M. Let $\Pi_t^{M,o}\left(y_j; \mu_Z, n_Z, \bar{y}_Z^D, \bar{y}_Z^F, Z\right)$ denote this amount.

In equilibrium, the following must hold

$$\text{for } M = D, F: \sum_{t=1}^{2} \Pi_t^M \left(y_j; \mu_Z, n_Z, \bar{y}_Z^D, \bar{y}_Z^F, Z \right) - F_M \geq 0 \text{ if } j \in I_M (Z); \tag{4}$$

$$\text{for } M = D, F: \sum_{t=1}^{2} \Pi_t^{M,o} \left(y_j; \mu_Z, n_Z, \bar{y}_Z^D, \bar{y}_Z^F, Z \right) - F_M \leq 0 \text{ if } j \notin I_D (Z) \cup I_F (Z); \tag{5}$$

$$\sum_{t=1}^{2} \left(\Pi_t^D \left(y_j; \mu_Z, n_Z, \bar{y}_Z^D, \bar{y}_Z^F, Z \right) - \Pi_t^{F,d} \left(y_j; \mu_Z, n_Z, \bar{y}_Z^D, \bar{y}_Z^F, Z \right) \right) \geq F_D - F_F \text{ if } j \in I_D (Z); \tag{6}$$

$$\sum_{t=1}^{2} \left(\Pi_t^{D,d} \left(y_j; \mu_Z, n_Z, \bar{y}_Z^D, \bar{y}_Z^F, Z \right) - \Pi_t^F \left(y_j; \mu_Z, n_Z, \bar{y}_Z^D, \bar{y}_Z^F, Z \right) \right) \leq F_D - F_F \text{ if } j \in I_F (Z). \tag{7}$$

2.3 The equilibrium definition

We are now ready to formally state the definition of the equilibrium. For any Z, the perfect Bayesian equilibrium of this game is a quadruple $\left\{ I_D (Z), I_F (Z), \left\{ x_{it}^D (N) \right\}_{N,i,t}, \left\{ x_{it}^F (N) \right\}_{N,i,t} \right\}$ which satisfies the following:

1. For each investor, $\left\{ x_{it}^D (N) \right\}_{i=1}^{N}$ and $\left\{ x_{it}^F (N) \right\}_{i=1}^{T-N}$ solve (1).

2. For each manager $j \in I_D (Z)$, conditions (4) and (6) are satisfied. For each manager $j \in I_F (Z)$, conditions (4) and (7) are satisfied. For each manager $j \notin I_D (Z) \cup I_F (Z)$, condition (5) is satisfied.

3. The beliefs satisfy $\phi_{j1}^M (N) = E [\alpha_j | y_j] = y_j$, $\sigma_{\alpha M1}^2 (N) = Var (\alpha_j | y_j) = \sigma_\eta^2$, and

$$\begin{aligned} \phi_{j2}^M (N) &= E \left[\alpha_j \mid \{ y_j \}_{j=1}^T, \left\{ R_{j1}^F \right\}_{j=1}^N, \left\{ R_{j1}^F \right\}_{j=1}^{T-N}, v_{D1} \right], \\ \sigma_{\alpha M2}^2 (N) &= Var \left(\alpha_j \mid \{ y_j \}_{j=1}^T, \left\{ R_{j1}^F \right\}_{j=1}^N, \left\{ R_{j1}^F \right\}_{j=1}^{T-N}, v_{D1} \right). \end{aligned}$$

Part (1) says that each investor maximizes her utility, given the funds she observes and the beliefs she holds. Part (2) says that when deciding which type of mutual fund, if any, to open, each manager maximizes her expected profit. Part (3) says that investors update their beliefs using Bayes' rule.

In the rest of the paper, we follow Burdett and Judd (1983) in that we look at the properties of the equilibria as $Z \to \infty$. To that end, we place the following assumption on how the distribution of signals $\{y_i\}_{i=1}^{Z}$ changes with Z.

Assumption C We assume that each manager's signal is drawn independently from a probability distribution function $H(\cdot)$ which is continuous and has a compact support.

2.4 Discussion of the assumptions

Before we move to solving the model, let us discuss its most important assumptions. (For the discussion of the remaining assumptions, see Section 5.)

It is crucial in our model that each investor observes only a finite number of mutual funds: as it shall become clear later, if investors observed an infinite number of foreign-asset funds, they could perfectly infer the foreign fundamentals, thereby undoing the informational asymmetry between the asset types. At the same time, the analysis of a finite economy in which $T = Z$ is intractable because atomic managers do not behave competitively. Allowing for $Z \to \infty$ assures that they do. Hence, if Z were finite and $T = Z$, our results would still hold if we additionally assumed that managers behave competitively. Since in reality there is only a finite number of mutual funds, this implies that our results are relevant even if investors have access to data about all of them. We want to point out, however, that we see $T < Z$ as a reasonable assumption even in a finite economy. The assumption that $T < Z$ can be justified on the grounds of bounded rationality, search frictions, or costly tracking and processing of information about mutual funds.[7]

We assume that managers use active management, which—depending on the managers'

[7] For bounded rationality justification see Merton's 1987 investor-recognition hypothesis, which has received wide empirical support; e.g., in Falkenstein, 1996; Fang and Peress, 2009; and Garcia and Norli, 2010.

abilities (α_j) and market fundamentals (v_{Mt})—may result in nonzero excess returns. The excess returns R_{tj}^{M} of manager j, specializing in assets of market M, are defined here with respect to a relevant passive benchmark that is easily observable by individual investors, for example, the S&P 500 index for U.S. assets or the Nikkei index for Japanese assets.

There is evidence (see, e.g., Ferson and Schadt, 1996) that publicly available information is statistically significant for the performance of mutual funds. In our model, the excess returns that can be obtained based on publicly available information about assets M are modeled by v_{Mt}. Assumption A is hence similar to the assumption of Van Nieuwerburgh and Veldkamp (2009) in that domestic individual investors have an informational advantage about the domestic economy. This assumption is motivated by the observation that individual investors listen to domestic media and talk to other individuals who might have expertise in domestic assets. They may learn about the performance of other investment vehicles, such as hedge funds, or of domestic private investors. Therefore, they can reasonably estimate what excess returns the mutual funds should have generated based on information available at the time.[8,9] Investors, however, have much less information about the foreign markets.

There is quite a bit of empirical evidence on the asymmetric information of individual investors. Portes et al. (2001) present empirical evidence that investment flows are positively correlated with physical proximity as well as with the information flow. Kang and Stulz (1997) show that foreign investment in Japanese equities is concentrated in the largest firms, which is consistent with the hypothesis that foreign investors have less information about small firms than local investors do. Shiller et al. (1996) conclude that expectations about market returns differ significantly between countries.

As in Chevalier and Ellison (1999), we interpret α_j as inherent stock-picking ability, timing

[8] Note that the assumption that investors know domestic fundamentals does not imply that they can easily replicate or undo the managers' trades that are based on public information. For example, our model requires only that investors know the domestic fundamentals ex post. For other reasons see Ferson and Schadt (1996).

[9] This assumption is also consistent with the evidence provided by Fang et al. (2011) that fund managers on average trade stocks with high media coverage.

ability, direct benefits from better education, and differences in the value of the social networks that different schools provide. Managers' stock-picking ability is related to the ability of the manager to acquire and process information about the likely prospects of individual assets. The signal y_j can be interpreted as the publicly available information about the manager, such as her curriculum vitae. Chevalier and Ellison (1999) showed that managers who attended higher-SAT undergraduate institutions systematically obtain higher risk-adjusted excess returns. They also cite a 1994 study by Morningstar, Inc., which discovered that "over the previous five years diversified mutual funds managed by "Ivy League" graduates had achieved raw returns that were 40 basis points per year higher than those of funds managed by non-Ivy League graduates" (reported in *Business Week*, July 4, 1994, p. 6).

Finally, we want to point out that although we formally analyze only the market of mutual funds that are available to domestic investors, there are also (unmodeled) funds registered in the foreign country which invest in the two asset types, but are available only to foreign investors. Although the foreign-asset market is unmodeled, the same results apply due to symmetry. The assumption that investors from the domestic and the foreign country have access to a disjoint pool of managers is an important one. It reflects existing legal constraints, by which most countries allow only the legal residents of a given country to invest in the mutual funds registered in that country. Each country has its own rules and regulations. U.S. investors can only buy funds that are registered with the Securities and Exchange Commission (SEC); mutual funds authorized for sale in Europe are governed by regulations from the Undertakings for Collective Investment in Transferable Securities (UCITS). Likewise, if you are a Hong Kong resident, your choice of funds would be limited to those regulated by Hong Kong's Mandatory Provident Funds Authority.

3 The Equilibrium Analysis

To highlight the role that our assumptions play, we analyze three versions of the model: a benchmark model in which there is no uncertainty about the ability of the managers (Section 3.1), an exogenous specialization model in which the abilities are uncertain but the allocation of managers to assets is fixed at the allocation from the benchmark model (Section 3.2.1), and the full model (Section 3.2.2). We delegate all proofs to the appendix, but start here with the derivation of the investors' capital allocation, as this will help in developing the intuition for the results.

Let $q_t^M(N) \equiv \sum_j x_{jt}^M(N)$ be the total amount of capital allocated by an investor with N in period t to the assets in market M. The solution to the investor's problem (1) is

$$x_{jt}^D(N) = \frac{q_t^D(N)}{N} + \frac{\phi_{jt}^D - \frac{1}{N}\sum_{i=1}^N \phi_{it}^D}{\gamma\left(\sigma_{\alpha Dt}^2 + \sigma_\varepsilon^2\right)} \text{ for } N > 0, \text{ and} \tag{8}$$

$$x_{jt}^F(N) = \frac{q_t^F(N)}{T-N} + \frac{\phi_{jt}^F - \frac{1}{T-N}\sum_{i=1}^{T-N} \phi_{it}^F}{\gamma\left(\sigma_{\alpha Ft}^2 + \sigma_\varepsilon^2\right)} \text{ for } N < T, \tag{9}$$

where

$$q_t^D(N) = \frac{\gamma\left(\sigma_v^2 + \frac{1}{T-N}\left(\sigma_{\alpha Ft}^2 + \sigma_\varepsilon^2\right)\right) + \left(\frac{1}{N}\sum_{i=1}^N \phi_{it}^D - \frac{1}{T-N}\sum_{i=1}^{T-N}\phi_{it}^F\right)}{\gamma\left(2\sigma_v^2 + \frac{1}{T-N}\left(\sigma_{\alpha Ft}^2 + \sigma_\varepsilon^2\right) + \frac{1}{N}\left(\sigma_{\alpha Dt}^2 + \sigma_\varepsilon^2\right)\right)} \text{ for } N > 0, \tag{10}$$

$$q_t^F(N) = \frac{\gamma\left(\sigma_v^2 + \frac{1}{N}\left(\sigma_{\alpha Dt}^2 + \sigma_\varepsilon^2\right)\right) - \left(\frac{1}{N}\sum_{i=1}^N \phi_{it}^D - \frac{1}{T-N}\sum_{i=1}^{T-N}\phi_{it}^F\right)}{\gamma\left(2\sigma_v^2 + \frac{1}{T-N}\left(\sigma_{\alpha Ft}^2 + \sigma_\varepsilon^2\right) + \frac{1}{N}\left(\sigma_{\alpha Dt}^2 + \sigma_\varepsilon^2\right)\right)} \text{ for } N < T. \tag{11}$$

Clearly, $x_{jt}^D(0) = q_t^D(0) = x_{jt}^F(T) = q_t^F(T) = 0$.

Equations (10) and (11) say that the amount of capital that an investor with N channels to domestic assets depends on the number of domestic-asset funds she observes, N; the average expected ability of the observed funds of each type, $\frac{1}{N}\sum_{i=1}^N \phi_{it}^D$ and $\frac{1}{T-N}\sum_{i=1}^{T-N}\phi_{it}^F$; and the uncertainty about managers' abilities, $\sigma_{\alpha Dt}^2$ and $\sigma_{\alpha Ft}^2$. Equations (8) and (9) say that the amount of capital this investor channels to a particular manager depends additionally on how the ability of this manager differs from the abilities of other managers of the same type. Note that the behavior of each investor does not depend on Z.

In what follows, for an equilibrium with the corresponding $\mu_Z, n_Z, \bar{y}_Z^D$, and $\bar{y}_Z^F$, we will be interested in the expected amount of capital channeled by a random investor to the assets M at time $t: Q_{Mt}\left(\mu_Z, n_Z, \bar{y}_Z^D, \bar{y}_Z^F, Z\right) \equiv \sum_{N=0}^{T} P\left(N|\mu_Z, n_Z, Z\right) E\left[q_t^M\left(N\right)|N, \bar{y}_Z^D, \bar{y}_Z^F, Z\right]$, where $P\left(N|\mu_Z, n_Z, Z\right)$ is the probability that a randomly selected investor observes N domestic-asset funds in an equilibrium with μ_Z and n_Z. We will say that a model exhibits home bias in period t, if $Q_{Dt} - \frac{1}{2} > 0$, which is equivalent to $Q_{Dt} - Q_{Ft} > 0$.

3.1 The benchmark model with no uncertainty about abilities

We start by analyzing a benchmark version of the model: we assume that investors face no uncertainty about managers' abilities, that is, $\sigma_\eta^2 = 0$. We maintain all other assumptions; in particular, the assumption that investors have asymmetric information about the fundamentals.

The following proposition describes the properties of the equilibria as Z becomes large. It states that investors' asymmetric information is not enough to generate any of the empirical regularities mentioned in the introduction. In particular, in the presence of mutual funds, individual asymmetric information does not lead to home bias.

Proposition 1 *If $\sigma_\eta^2 = 0$ and $F_D = F_F$, then*

i) the equilibrium fraction of managers specializing in each market is approximately the same, i.e.,

$$\lim_{Z\to\infty}\left|n_Z - \frac{1}{2}\right| = 0 \text{ with probability } 1;$$

ii) the expected average ability of managers specializing in each asset type is approximately the same, i.e.,

$$\lim_{Z\to\infty}\left|\bar{y}_Z^D - \bar{y}_Z^F\right| = 0 \text{ with probability } 1;$$

iii) the expected amount of capital invested in each asset type is approximately the same, i.e.,

$$\lim_{Z\to\infty}\left|Q_{Dt}^{Benchmark}-\frac{1}{2}\right|=0 \textit{ for } t\in\{1,2\} \textit{ with probability } 1.$$

The intuition for the above equilibria is as follows.[10] Since there is no uncertainty about the ability of the managers, investing via each of them is equally risky. When $\bar{y}_Z^D = \bar{y}_Z^F$, then the capital channeled to domestic assets brings on average the same excess returns as the capital channeled to foreign assets. And when additionally $n_Z = \frac{1}{2}$, the idiosyncratic risk ε_{it} can be diversified to the same extent for domestic and foreign investments. Hence, if $\bar{y}_Z^D = \bar{y}_Z^F$ and $n_Z = \frac{1}{2}$, investors find it profitable to channel the same amount of capital to both asset types. This all means that no matter which assets a manager specializes in, she directly competes with the same number of funds, with funds of the same ability, and for the same amount of capital. Hence, she is indifferent between both asset types, and this supports the equilibrium.[11]

3.2 Two models with uncertainty about abilities

In the next two sections, we restore the assumption that investors face uncertainty about managers' abilities, i.e., $\alpha_j = y_j + \eta_j$ with $\sigma_\eta^2 \neq 0$. In this case, investors' beliefs about managers' abilities can change over time as described in part (3) of the equilibrium definition. We now derive the evolution of these beliefs.

Part (3) of the equilibrium definition says that $\phi_{j1}^M(N) = y_j$, $\sigma_{\alpha M1}^2(N) = \sigma_\eta^2$, and that in the second period investors condition their beliefs on $\{y_j\}_{j=1}^T$, $\left\{R_{j1}^F\right\}_{j=1}^N$, $\left\{R_{j1}^F\right\}_{j=1}^{T-N}$, and v_{D1}. Since $R_{tj}^M = \alpha_j + \varepsilon_{tj} + v_{Mt}$, and ε_{tj} is independent across managers and periods, it follows

[10] Note that Proposition 1 does not place any restrictions on $I_D(Z)$ and $I_F(Z)$ other than that they must result in $\mu_Z, n_Z, \bar{y}_Z^D$, and $\bar{y}_Z^F$ that satisfy the properties outlined there. Hence, there may be multiple equilibria.

[11] It is more difficult to see why other equilibria cannot exist. A natural candidate for an equilibrium would be $n_Z > \frac{1}{2}$ and $\bar{y}_Z^D < \bar{y}_Z^D$. In such an equilibrium, the incentive to leave the domestic asset market due to its being overcrowded would be balanced by the incentive to stay in it due to a lower quality of the direct competitors. The proof of Proposition 1, however, shows that when $n_Z > \frac{1}{2}$, then actually the most able managers have the highest incentive to invest domestically.

that when updating the beliefs about a domestic-asset manager j, the excess returns of other managers do not carry any information about the ability of manager j. Therefore,

$$\phi_{j2}^{D}(N) = E\left[\alpha_j \mid y_j, R_{j1}^{D}, v_{D1}\right];$$

$$\sigma_{\alpha D2}^{2}(N) = Var\left(\alpha_j \mid y_j, R_{j1}^{D}, v_{D1}\right) = \sigma_{\varepsilon}^{2}\frac{\sigma_{\eta}^{2}}{\sigma_{\varepsilon}^{2}+\sigma_{\eta}^{2}}. \quad (12)$$

When updating her beliefs about a foreign-asset manager j, however, the investor does not observe the foreign fundamentals. Since foreign fundamentals affect the excess returns of all foreign-asset funds, the excess returns of all foreign-asset funds carry some information about manager j's ability. Therefore,

$$\phi_{j2}^{F}(N) = E\left[\alpha_j \mid \{y_j\}_{j=1}^{T-N}, \left\{R_{j1}^{F}\right\}_{j=1}^{T-N}\right];$$

$$\begin{aligned}\sigma_{\alpha F2}^{2}(N) &= Var\left(\alpha_j \mid \left\{y_j^{F}\right\}_{j=1}^{T-N}, \left\{R_{j1}^{F}\right\}_{j=1}^{T-N}\right) = \quad (13)\\ &= \frac{\sigma_{\eta}^{2}}{\sigma_{\varepsilon}^{2}+\sigma_{\eta}^{2}}\frac{\sigma_{\varepsilon}^{4}+\sigma_{\eta}^{2}\left(\sigma_{v}^{2}+\sigma_{\varepsilon}^{2}\right)+(T-N)\,\sigma_{v}^{2}\sigma_{\varepsilon}^{2}}{\sigma_{\varepsilon}^{2}+\sigma_{\eta}^{2}+(T-N)\,\sigma_{v}^{2}}.\end{aligned}$$

Let us now make two observations that will play a crucial role in the rest of the analysis. First, $E\left[\phi_{jt}^{M}(N)\,|y_j\right] = y_j$; hence, at the beginning of the game, the signals about managers' ability are the best predictors of what beliefs investors will have about the managers as the game proceeds. Second, $\sigma_{\alpha F2}^{2}(N) > \sigma_{\alpha D2}^{2}(N)$ for any N. That is, the asymmetric information about fundamentals results in investors facing higher uncertainty about the ability of foreign-asset managers than about the ability of domestic-asset managers.

3.2.1 The exogenous specialization model

In this section, we assume that managers are allocated to the asset types as in the limit economy from Proposition 1 of the benchmark model; that is, $n_Z = \frac{1}{2}$ and $\bar{y}_Z^D = \bar{y}_Z^F$. In this way we can isolate the direct effect of Assumptions A and B on capital allocation across assets.[12]

Since $\sigma^2_{\alpha F2} > \sigma^2_{\alpha D2}$, in the second period investors can identify highly able managers more precisely in the domestic-asset market than they can in the foreign-asset market. This makes domestic investments less risky. Proposition 2 states that as a result, investors find it optimal to channel more of their capital to domestic investments.

Proposition 2 *If $F_D = F_F$, and we allocate the managers so that $n_Z = \frac{1}{2}$ and $\bar{y}_Z^D = \bar{y}_Z^F$, then for Z large enough,*

i) $Q_{D1}^{Ex\ Spec} = \frac{1}{2}$;

ii) $Q_{D2}^{Ex\ Spec} > \frac{1}{2}$.

Proposition 2 states that the combination of asymmetric information and uncertainty about the managers' abilities generates asymmetry in capital allocation across assets in the second period.

3.2.2 The endogenous specialization model

In this section we analyze the full version of the model. In particular, managers choose optimally in which assets, if any, to specialize.

Proposition 2 says that if $n_Z = \frac{1}{2}$ and $\bar{y}_Z^D = \bar{y}_Z^F$, then in the second period investors channel more capital to domestic assets. This increases managers' incentives to open domestic-asset

[12] Technically, for small Z, it may be impossible to allocate managers to assets in a way that results in $\bar{y}_Z^D = \bar{y}_Z^F$. However, as $Z \to \infty$, this is possible with probability 1.

mutual funds. It turns out that it is the best managers that have the highest incentive to do so, which leads to the following equilibrium in the full model:

Proposition 3 *In any equilibrium, if $F_D = F_F$, then*

i) the expected average ability of domestic-asset managers is higher:

$$\lim_{Z\to\infty} \left(\bar{y}_Z^D - \bar{y}_Z^F\right) > 0 \text{ with probability } 1;$$

ii) the average market value of the domestic-asset funds is higher:

$$\lim_{Z\to\infty} \left(\frac{\sum_{t=1}^{2} Q_{Dt}^{FullModel}}{n_Z} - \frac{\sum_{t=1}^{2} Q_{Ft}^{FullModel}}{1-n_Z} \right) > 0 \text{ with probability } 1;$$

iii) there exist distribution functions H satisfying Assumption C, and an open set of parameters, such that in some equilibrium, $\lim_{Z\to\infty} n_Z < \frac{1}{2}$ and $\lim_{Z\to\infty} \left(Q_{Dt}^{FullModel} - \frac{1}{2}\right) > 0$ for $t \in \{1,2\}$ with probability 1.[13]

Proposition 3 states that our model accounts for two of the empirical regularities about the mutual fund industry described in the introduction, and under certain conditions it accounts for all three. Part (*ii*) says that the average amount of capital managed by each domestic-asset fund is larger than the average amount of capital managed by each foreign-asset fund. Part (*i*) suggests that this is due to the fact that the domestic-asset funds are on average of higher ability. Part (*ii*) also implies that either $Q_{Dt}^{FullModel} > Q_{Ft}^{FullModel}$ for $t \in \{1,2\}$, or $n_Z < \frac{1}{2}$, or both. That is, either more capital is channeled to domestic assets, or there are fewer domestic-asset mutual funds, or both. Part (*iii*) states that one can find some distributions of ex-ante ability for which $n_Z < \frac{1}{2}$ and $Q_{Dt}^{FullModel} > Q_{Ft}^{FullModel}$ for $t \in \{1,2\}$.

[13] Multiple simulations show that for all parameters one can find functions H with the desirable properties, but we were unable to show this analytically.

The intuition for Proposition 3 is as follows. As established before, investors learn about the abilities of domestic-asset managers more precisely than about the abilities of foreign-asset managers. This makes investments in domestic assets less risky, but also makes the investors respond more strongly to the perceived differences in domestic-asset managers' abilities (see the second part of equations (8) and (9)). That is, ability is rewarded more in the domestic-asset market; hence, this market attracts higher-ability managers. This in turn leads to $\bar{y}_Z^D > \bar{y}_Z^F$. To understand part (ii), note that in equilibrium, domestic-asset managers receive more capital than they would if they invested in foreign assets. Since they are also of higher quality on average, they must be receiving more capital than the foreign-asset managers.

The reason why Proposition 3 does not characterize n_Z fully is that the equilibrium n_Z depends on the distribution from which the ex-ante abilities are drawn, H. To understand why, consider the best foreign-asset manager. If $n_Z < \frac{1}{2}$, this manager faces two competing forces. On the one hand, if $n_Z < \frac{1}{2}$, then fewer managers compete for the capital channeled to domestic assets than for the capital channeled to foreign assets. This creates an incentive for the best foreign-asset manager to deviate to domestic assets. On the other hand, from part (i) we know that $\bar{y}_Z^D > \bar{y}_Z^F$. This means that the current competitors of this manager are of lower ability than the competitors she would face, if she deviated to domestic assets. Hence, she currently receives a higher fraction of the capital channeled to the assets she specializes in than she would if she deviated to the domestic assets. Which effect dominates depends on the ex-ante distribution of abilities. If there is no ex-ante heterogeneity across managers, then necessarily $\bar{y}_Z^D = \bar{y}_Z^F$, and the second effect disappears. In such cases, n_Z must increase. If the ex-ante heterogeneity across managers is quite large, however, the second effect may dominate, and $n_Z < \frac{1}{2}$ may be sustained in equilibrium.

When $n_Z > \frac{1}{2}$, then part (ii) implies that $Q_{Dt}^{FullModel} > Q_{Ft}^{FullModel}$ for $t \in \{1, 2\}$. When $n_Z < \frac{1}{2}$, however, the impact of $\bar{y}_Z^D > \bar{y}_Z^F$ and the impact of $n_Z < \frac{1}{2}$ on individual investments go in opposite directions. If $\bar{y}_Z^D > \bar{y}_Z^F$, investors expect on average higher excess returns from domestic investments; this incentivizes them to channel more capital to domestic assets. If $n_Z < \frac{1}{2}$, then most investors will observe $N < T - N$. Those investors will be able to diversify the idiosyncratic risk ε_{tj} of managers better for their foreign investments; this incentivizes them to channel capital to foreign assets. Which effect dominates can vary, but part (*iii*) of Proposition 3 states that there are certain distributions of ex-ante ability for which more capital is channeled to domestic assets. In Section 4, we solve the model numerically and show that this happens for reasonable sets of parameters.

Before we conclude this section, let us state the following result about the behavior of individual managers in the limit at $Z = \infty$.

Remark 1 *In the limit at* $Z = \infty$, *there exists a threshold* $\hat{y}_o$, *such that if* $y_j < \hat{y}_o$, *then* $y_j \notin I_D(\infty) \cup I_F(\infty)$. *Moreover, one of the following holds:*

a. There exists a threshold $\hat{y}_d$, *such that if* $y_j > \hat{y}_d$, *then* $y_j \in I_D(\infty)$, *and if* $y_j \in (\hat{y}_o, \hat{y}_d)$, *then* $y_j \in I_F(\infty)$.

b All managers who operate are indifferent between specializing in domestic and foreign assets.[14]

4 Numerical Analysis

In this section, we solve the model numerically. This exercise enables us to study the magnitude of the established effects. Because there are no conventional parameter values for this type of model, we assume parameters that are frequently used in the literature.[15]

[14] From Proposition 3 and Remark 1 one can see that the equilibrium relationship between managers' abilities and their entry and specialization decisions is similar to the one obtained by Melitz (2003). Melitz (2003) looks at the impact of trade on the industry structure and shows that most productive firms export, less productive firms do not, and the least productive firms exit the market. The mechanism behind this result is different, however, than in our paper and relies on the fixed cost of entry to one market.

[15] We could conduct a proper calibration, but this model is too stylized for such exercise to have much value.

Using the values that Dang, Wu, and Zechner (2008) calibrated using the CRSP survivor-bias-free U.S. mutual fund database (1961 to 2002), we set the volatility of the tracking error, $\sigma_\nu + \sigma_\varepsilon$, to 10%. As in Van Nieuwerburgh and Veldkamp (2009), we consider a 10% initial information advantage, which implies that the standard deviation of fundamentals, σ_ν, is set to 1% and the standard deviation of the idiosyncratic risk, σ_ε, is set to 9%. Dang, Wu, and Zechner (2008) calibrate the variance of the ability of a fund manager to be 0.04; hence, we set $\sigma_\eta = 0.2$. As Wei (2007) notes, the ICI's Mutual Fund Fact Book indicates that a typical household holds four mutual funds; hence, we set $T = 4$. The coefficient of risk aversion is $\gamma = 1$. We solve the model as the size of the economy grows large; hence, we drop subscript Z from the relevant variables. We assume that $H(y_j) = \frac{1}{2} - \frac{q}{e} + \frac{y_j}{e}$ with $y_j \in \left[q - \frac{1}{2}e, q + \frac{1}{2}e\right]$ and $e > 0$. With this formulation, the expected ability of potential managers is equal to q, and e measures the difference between the maximum and the minimum ability. We set $q = 0.04$ and $e = 0.08$. In equilibrium, only the best managers open mutual funds; hence, if μ managers operate, then these are managers $j \in [0, \mu]$. Finally, we assume that $F_F = (1+g) F_D$ and solve for the equilibrium for different values of g. We set $F_D = 2.8$ in order to have the equilibrium ratio of the size of an average fund over the minimum fund size be equal to 4.7 for $g = 0$, as calibrated by Berk and Green (2004). The numerical results are independent of the fundamentals in the economy, $\bar{\nu}$; hence, we do not need to take a stand on the debate over the relative performance of fund managers with respect to passive benchmarks.

4.1 Results

In this section we show that our model accounts for the three salient features of the data reported by Chan, Covrig, and Ng (2005) and Hau and Rey (2008), which are: (1) There is equity home bias at the fund level. (2) On average, the number of foreign-asset funds is larger than the number of domestic-asset funds. (3) On average, the market value of foreign-asset funds is smaller than the market value of domestic-asset funds.

The solid lines in Figure 1 represent the equilibrium in the full model as a function of g. Panel A plots the fraction of operating managers who invest domestically, n; Panel B plots the difference in average ability, $\bar{y}^D - \bar{y}^F$; Panel C, reports the ratio of the market value of domestic-asset funds to the market value of foreign-asset funds $\phi \equiv \frac{(Q_{D1}+Q_{D2})/n}{(Q_{F1}+Q_{F2})/(1-n)}$; Panel D shows the fraction of operating managers, μ; and Panel E plots the amount of capital invested in the domestic-asset market in the second period, Q_{D2}. The dashed lines represent the same variables for the benchmark model.[16]

Let us start by describing the results for $g = 0$. Recall that in the benchmark model, investors have perfect information about managers' abilities. Consistent with Proposition 1, for the benchmark model the number of funds operating, the average fund manager's ability, and the amount of attracted capital are the same in both markets, and the ratio of the market value of domestic-asset to foreign-asset funds is 1. Consistent with Proposition 3, Panel B of Figure 1 shows that in the full model the average ability of domestic-asset managers is higher—specifically, by 46 basis points—than the average ability of foreign-asset managers. Also as predicted by Proposition 3, Panel C of Figure 1 shows that domestic-asset mutual funds have a higher market value on average: the ratio of the market value of domestic-asset to foreign-asset funds is $\phi = 1.76$. Proposition 3 was mute on n, but Panel A shows that for our parameters, only 34% of operating managers specialize in domestic assets; that is, the number of domestic-asset funds is lower than the number of foreign-asset funds. Nevertheless, as Panel E shows, domestic assets attract more capital: the total expected amount of capital invested in domestic assets in the second period is $Q_{D2}^{Full\ Model} = 54.9\%$. Summing up, even when the cost of opening both types of mutual funds is the same, our model can qualitatively account for all three stylized facts reported above.

However, the second-period home bias generated by the model when $g = 0$ is not very high.

[16] We do not report Q_{D1}. Since asymmetric information plays no role in the first period, the first-period results are not representative of what one should expect empirically.

This is because $n = 34\%$, and although the domestic-asset managers generate higher excess returns on average, investors cannot effectively diversify the idiosyncratic risk ε_{it} associated with these managers. As a result, domestic investments are less attractive to investors than the difference $\bar{y}^D - \bar{y}^F$ would suggest. However, at $g = 0$, the model is likely to underestimate n. In practice, g is likely to be positive.[17] Indeed, Panel E shows that as g increases, n and $\bar{y}^D - \bar{y}^F$ weakly increase, and $Q_{D2}^{Full\ Model}$ increases. Moreover, $Q_{D2}^{Full\ Model}$ increases more than $Q_{D2}^{Benchmark}$.

Let us now explain the intuition for the way our results vary with g. It turns out that in both models, when $g = 0$, all managers are indifferent between investing in domestic and foreign assets. In the equilibrium of the full model, domestic-asset funds on average are better than foreign-asset funds, but that is not uniformly true. As g increases, opening a foreign-asset fund becomes less attractive in both models. To restore the equilibrium, either n must increase, making investing in domestic assets more competitive, or the average quality of domestic-asset managers must increase, making the competitors in domestic assets stronger. Panels A and B of Figure 1 show that for the benchmark model, the latter happens. For the full model, Panel A shows that the latter happens for small values of g : for $g < 0.5\%$, n is constant at 34%. For g around 0.5%, however, domestic-asset managers are uniformly better than foreign-asset managers. Hence, as g increases further, n has to adjust to make foreign assets more attractive.

According to Chan et al. (2005) and Hau and Rey (2008), the actual fraction of domestic-asset funds is 46%. Panel A shows that to match this number, the cost of specializing in foreign assets must be 1.5% higher than the cost of specializing in domestic assets, that is, $g = 1.5\%$. Panels C and E show that if $g = 1.5\%$, then $\phi = 2.2$, and $Q_{D2}^{Full\ Model} = 76\%$. Notice that a very small fixed-cost differential allows us to match the fraction of domestic-asset funds

[17] See Bonser-Neal et al. (1990); Hardouvelis et al. (1994); Claessens and Rhee (1994); and Errunza and Losq (1985) for empirical support on investment barriers to international markets.

to the data and generates a sizeable home bias. The key reason for this substantial amount of home bias is $\bar{y}^D - \bar{y}^F$. This numerical analysis implies that managers investing domestically generate excess returns between 46 and 60 basis points higher than those investing in foreign assets (see Panel B). This difference in ability between fund managers is not unreasonable, as it is a magnitude similar to the numbers reported by Chevalier and Ellison (1999) and the 1994 study by Morningstar, Inc. (reported in *Business Week*, July 4, 1994, p. 6) for the difference in excess returns between Ivy League graduates and non-Ivy League graduates.

It is important to point out that the benchmark model is not able to generate a sizeable home bias even if the fixed-cost differential is high. In Panel E, we can see that an increase in foreign fixed costs leads to only a modest increase in the amount of capital allocated to the domestic assets. Panel A shows that for any reasonable g, the benchmark model is inconsistent with the stylized fact that there are fewer mutual funds investing domestically.

To show what fraction of home bias is driven directly by the combination of asymmetric information and uncertainty about managers' abilities, and what fraction by managers' responses to that, in Panel F we depict separately $Q_{D2}^{Ex\ Spec} - Q_{D2}^{Benchmark}$ (dashed line) and $Q_{D2}^{Full\ Model} - Q_{D2}^{Ex\ Spec}$ (solid line). Panel F shows that the home bias generated directly, $Q_{D2}^{Ex\ Spec} - Q_{D2}^{Benchmark}$, is negligible; most of the home bias comes from the fact that managers respond optimally to the expected asymmetry in investors' capital allocation.

Interestingly, for $g > 0$, $Q_{D2}^{Ex\ Spec} - Q_{D2}^{Benchmark}$ may be negative (Panel F). This means that if we fix the allocation of managers to asset types, introducing the uncertainty about managers' abilities may even generate negative home bias. This may sound surprising at first, but the reason for this is as follows. When the uncertainty about abilities is introduced in the exogenous specialization model, there are two effects. The first effect, which appeared in Proposition 2 for $g = 0$, is that investors update their beliefs about the abilities of foreign-asset managers more slowly, and this causes them to channel less capital to foreign assets. The second effect appears because for $g > 0$, both models have $\bar{y}^D > \bar{y}^F$. In the benchmark model, investors react to this by channelling more capital to domestic-asset managers. In the exogenous specialization model, however, investors are uncertain about the abilities; hence, they react less strongly to this ability differential. Panel F of Figure 1 shows that for sufficiently high g, the latter effect dominates.

4.2 Comparative Statics

In this section we set $g = 0$ and investigate the role of different parameters of the model. Figures 2 to 5 show, respectively, that we observe higher home bias in the second period when (i) the ex-ante heterogeneity of managers is not too large; (ii) unobserved heterogeneity is large; (iii) uncertainty about the fundamentals is not too large; and (iv) the idiosyncratic shocks to managers' excess returns are not too large.

Figure 2 shows that home bias decreases with the ex-ante heterogeneity of managers, e. When the ex-ante heterogeneity is large, the best managers are much better than the mediocre ones. That makes it more difficult for the mediocre ones to compete with the best ones in the domestic-asset market; therefore, more of them invest in foreign assets. As a result, investors can better diversify the idiosyncratic element ε_{tj} of the foreign-asset managers, and that makes foreign investments more attractive, decreasing home bias. For any e, however, there is home bias of at least 5 percentage points.

Figure 3 shows how the equilibrium varies with the unobserved heterogeneity of managers, σ_η. When the unobserved heterogeneity is large, the speed of learning is important, and the information asymmetry across markets plays a bigger role. An increase in heterogeneity leads to an increase in home bias in the second period.

Figure 4 shows that home bias decreases with uncertainty about fundamentals, σ_v. Here, we have two competing effects. Uncertainty about fundamentals increases the informational disadvantage of the foreign-asset market, and therefore leads to higher home bias. However, it also increases the need for cross-market diversification, which pushes investors to distribute their capital evenly across the markets. As we see in Figure 5, the latter effect dominates.

Figure 5 shows that the second-period home bias decreases with the manager-specific risk, σ_ε, but this finding is not robust to different values of e.[18] This is because an increase in manager-specific risk affects the equilibrium via three different channels. First, as σ_ε increases, it becomes more important for investors to diversify the idiosyncratic risk ε_{tj} of each manager's excess returns than to obtain high excess returns. As a result, managers' ability matters less, and this decreases home bias. Second, as σ_ε increases, the manager who was indifferent between the domestic-asset and foreign-asset markets before, now finds it profitable to enter the domestic-asset market. This is because the threshold manager prefers to enter a less crowded market, but previously she was deterred by the high ability of the competitors in the domestic-asset market. Since now ability matters less, she strictly prefers to enter the domestic-asset market. Therefore, n increases, and home bias increases. And finally, since an additional operating manager provides high diversification benefits, and hence attracts more capital, operating becomes profitable for lower-ability managers. Since the new, low-ability managers enter the foreign-asset market, the difference in average ability across markets increases, which increases home bias. The higher e is, the stronger this effect.

5 Discussion of the remaining assumptions

The assumption that managers can specialize in only one market is clearly a simplification, but it can be justified on many grounds. First, it might be disadvantageous for a manager to invest in many markets because there might be returns to scale in information processing, as suggested in Van Nieuwerburgh and Veldkamp (2009).[19] Another theoretical support for the existence of funds with narrow mandates is provided by He and Xiong (2008) in a model of

[18] Results available from the authors upon request.

[19] This assumption could be theoretically rationalized with a model similar to the one proposed by Van Nieuwerburgh and Veldkamp (2009). Their model shows that there are increasing returns to scale to information processing when investors have both a portfolio choice and an information processing choice.

delegated asset management in multiple markets with agency frictions. And finally, empirical evidence provided by Hau and Rey (2008) shows that the distribution of the markets in which mutual funds invest is bimodal. The distribution has a peak for completely home-biased funds and a peak for funds investing only in foreign assets.

In our model, investors cannot decide on the composition of T: T is drawn uniformly, and hence N is linked to n_Z. In practice, however, T and N are likely to be determined by investors who weigh the benefit of an extra fund against the cost of researching and tracking it. The effect of relaxing the assumption on T and on the composition of T is ambiguous. On the one hand, observing an additional foreign-asset fund increases the informational gain. This may lead investors to increase the number of observed foreign-asset funds, which would weaken our results. On the other hand, investors realize that they are better able to evaluate the ability of domestic-asset funds, which may prompt them to observe more domestic-asset mutual funds. Moreover, in our equilibrium a randomly drawn domestic-asset fund is likely to be of better ability than a randomly drawn foreign-asset fund. As a result, the investors may prefer to observe even more domestic-asset funds, which would strengthen our results.

One significant shortcoming of this paper is the fact that the fees charged by the managers are exogenous. Although this is definitely not completely realistic, we make this assumption in the interest of simplicity. Endogenizing the fee would not affect the fact that learning happens faster for the managers investing domestically and that this allows the investors to better allocate their investments. Ability would still be rewarded more in the domestic-asset market; hence, better-ability managers would still invest domestically. However, endogenizing the fee could weaken the result on the relative size of mutual funds: managers with higher ability could charge higher fees, decreasing the amount of capital under their management. However, the standard economic intuition on optimal pricing suggests that better managers would not raise prices to the level at which the demand for their services is identical to the demand for services of lower-ability managers.

6 Relationship to empirical literature and conclusions

This paper suggests that both asymmetric information at the individual level and uncertainty about the ability of portfolio managers play an important role in the delegated management industry. In our model, we show that these assumptions can explain a range of empirical observations documented in French and Poterba (1991), Tesar and Werner (1995), Chan et al. (2005), and Hau and Rey (2008). First, even if professional mutual fund managers are equally well informed about all markets, individual investments exhibit home bias. Second, the number of foreign-asset funds may be larger than the number of domestic-asset funds. And finally, the market value of foreign-asset funds is on average smaller than the market value of domestic-asset funds.[20]

Our model also delivers a sharp and testable prediction about the difference in abilities between managers investing domestically and internationally. A number of studies have compared performance of foreign and domestic institutional investors and found mixed evidence.[21] However, these studies compare the performance of managers who invest in the same market, for example Indonesia, but have different countries of origin and serve different investors, for example, Indonesian and U.S. investors. Our model predicts that the performance of managers from the same country differs depending on the market in which they invest; for example, U.S. mutual funds investing in domestic equities should have higher ability than U.S. mutual funds investing internationally.

We model only the market for delegated management in the domestic country, but clearly an analogous market exists in the foreign countries. The mutual fund market in the foreign

[20] One could point out an alternative and simpler explanation for the last two findings. It could be argued that due to returns to specialization, managers invest in only a subgroup of foreign markets. Since there are many foreign markets and only one domestic market, we should observe fewer mutual funds investing domestically. We find this explanation plausible, but not completely satisfactory. The reason for this is that in fact the level of specialization of funds investing in domestic assets seems to be higher than the level of specialization of funds investing in foreign assets.

[21] Dvorak (2005) finds that domestic investors outperform foreign investors in Indonesia, while Grinblatt and Keloharju (2000) and Seasholes (2000) find contradictory evidence in Finland and Taiwan (see also Choe, Kho, and Stulz, 2005).

countries should also have better managers investing in its own assets. However, managers in the foreign countries may draw from a different pool of talent; hence, we cannot conclude anything about the difference in abilities of managers who invest in the same assets but have a different country of origin. Only if managers in each country draw from the same pool of talent does our model suggest that the returns on investments in assets of one country are higher when executed by managers from this country than when executed by managers from another country. Two studies that compare investors from quite similar countries are Shukla and van Inwegen (1995) and Hau (2001). Shukla and van Inwegen (1995) look at U.K. and U.S. money managers investing in the U.S. equity market and Hau (2001) looks at sophisticated European investors investing in German equity. Both studies find that domestic investors outperform foreigners and hence are in line with our theoretical finding.

Another study that can be interpreted in favor of our last finding is Coval and Moskowitz (2001). Coval and Moskowitz (2001) look at U.S. mutual funds investing domestically, and show that they tend to perform better on local stocks. Moreover, managers with a stronger local bias perform better overall. While Coval and Moskowitz (2001) interpret this as evidence that portfolio managers may have an informational advantage in the local market, our model provides an alternative explanation. Managers may invest locally not to exploit their informational advantage—which they may not even have—but to respond optimally to the informational advantage of their clients. Moreover, we show that better managers have more incentive to invest locally, and as a result, managers who invest locally perform better.[22]

However, we believe that further research is needed to determine whether fund managers specializing in domestic assets have higher stock-picking ability than fund managers specializing in foreign assets. We hope this paper will encourage such research.

[22] For our interpretation to be plausible, it should be the case that the clients of these fund managers are also geographically localized. Indeed, Coval and Moskovitz (1999) find that clients exhibit a strong preference for local managers.

7 Appendix

The appendix is organized as follows. First, we derive the formulas for the profits of managers. After that, in Section 7.2.1 we define an auxiliary system of equations. In Section 7.2.2, we show that for any equilibrium with the corresponding quadruple $\left\{\mu_Z, n_Z, \bar{y}_Z^D, \bar{y}_Z^F\right\}$, as Z increases, the quadruple $\left\{\mu_Z, n_Z, \bar{y}_Z^D, \bar{y}_Z^F\right\}$ converges almost surely to one of the solutions of the auxiliary system of equations. Let $\left\{\mu_*, n_*, \bar{y}_*^D, \bar{y}_*^F\right\}$ denote this solution. Hence, when we prove propositions 1, 2, and 3, it will suffice to prove only the properties of $\left\{\mu_*, n_*, \bar{y}_*^D, \bar{y}_*^F\right\}$. Before we do that, however, in Section 7.3 we prove lemmas establishing some properties of the auxiliary system of equations that will be used later.

7.1 Derivation of $\Pi_t^{M,o}$ and a useful definition

Fix Z and some allocation of managers to assets $I_D(Z)$ and $I_F(Z)$ with the corresponding $\left(\mu_Z, n_Z, \bar{y}_Z^D, \bar{y}_Z^F\right)$. In this allocation we have $\mu_Z n_Z Z$ domestic-asset funds and $\mu_Z(1-n_Z)Z$ foreign-asset funds, with $\mu_Z Z$ funds in total. If $j \notin I_D(Z) \cup I_F(Z)$ deviates to opening a domestic-asset fund, then the probability that a random investor draws j and $N-1$ other domestic-asset managers is $P_{D,o}(N|\mu_Z, n_Z, Z_Z) = \frac{\binom{\mu_Z n_Z Z}{N-1}\binom{\mu_Z(1-n_Z)Z}{T-N}}{\binom{\mu_Z Z}{T}}$. The probabilities for $M = F$ are derived analogously. Using the above, we obtain:

$$\Pi_t^{M,o}\left(y_j; \mu_Z, n_Z, \bar{y}_Z^D, \bar{y}_Z^F, Z\right) = \sum_{N=0}^{T} Z P_{M,o}(N|\mu_Z, n_Z, Z)\, E\left[x_{jt}^M | y_j, N, \bar{y}_Z^D, \bar{y}_Z^F, Z\right]. \tag{14}$$

In what follows, we will also need the probability that in an allocation $I_D(Z)$ and $I_F(Z)$, a randomly selected investor observes N domestic-asset funds. This is $P(N|\mu_Z, n_Z, Z) = \frac{\binom{\mu_Z n_Z Z}{N}\binom{\mu_Z(1-n_Z)Z}{T-N}}{\binom{\mu_Z Z}{T}}$.

When taking the expectation of the formulas for the capital attracted by $j \in I_D(Z)$, equations (8) and (10), we have to take into account that j is one of the domestic-asset funds observed by her customers; hence, $E\left[\frac{1}{T-N}\sum_{i=1}^{T-N} \phi_{it}^F | y_j, N, y^D, \bar{y}^F, Z\right] = \bar{y}^F$, but we have that $E\left[\frac{1}{N}\sum_{i=1}^{N} \phi_{it}^D | y_j, N, y^D, \bar{y}^F, Z\right] = \frac{1}{N} y_j + \frac{(N-1)}{N} E\left[y_i | i \in I_D(Z) \setminus \{j\}\right] = \frac{1}{N} y_j + \frac{N-1}{N}\frac{n\mu Z \bar{y}^D - y_j}{n\mu Z - 1}$. Therefore,

$$E\left[x_{jt}^D | y_j, N, y^D, \bar{y}^F, Z\right] \equiv \frac{\gamma\left(\sigma_v^2 + \frac{1}{T-N}\left(\sigma_{\alpha F t}^2 + \sigma_\varepsilon^2\right)\right) + \frac{1}{N} y_j + \frac{N-1}{N}\frac{n\mu Z \bar{y}^D - y_j}{n\mu Z - 1} - \bar{y}^F}{\gamma N\left(2\sigma_v^2 + \frac{1}{T-N}\left(\sigma_{\alpha F t}^2 + \sigma_\varepsilon^2\right) + \frac{1}{N}\left(\sigma_{\alpha D t}^2 + \sigma_\varepsilon^2\right)\right)} + \frac{N-1}{N}\frac{y_j - \frac{n\mu Z \bar{y}^D - y_j}{n\mu Z - 1}}{\gamma\left(\sigma_{\alpha D t}^2 + \sigma_\varepsilon^2\right)}. \tag{15}$$

Analogously, using (11), and (9), we obtain:

$$E\left[x_{jt}^{D}|y_j,N,y^D,\bar{y}^F,Z\right]\equiv\frac{\gamma\left(\sigma_v^2+\frac{1}{N}\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)\right)+\frac{T-N-1}{T-N}\frac{(1-n)\mu Z\bar{y}^F-y_j}{(1-n)\mu Z-1}+\frac{1}{T-N}y_j-\bar{y}^D}{\gamma\left(T-N\right)\left(2\sigma_v^2+\frac{1}{T-N}\left(\sigma_{\alpha Ft}^2+\sigma_\varepsilon^2\right)+\frac{1}{N}\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)\right)}+\frac{T-N-1}{T-N}\frac{y_j-\frac{(1-n)\mu Z\bar{y}^F-y_j}{(1-n)\mu Z-1}}{\gamma\left(\sigma_{\alpha Ft}^2+\sigma_\varepsilon^2\right)}. \tag{16}$$

Finally, we will use the following definition:

Definition (1) *Let $\Delta_t^D\left(y_j;\mu,n,\bar{y}^D,\bar{y}^F,Z\right)$ and $\Delta_t^F\left(y_j;\mu,n,\bar{y}^D,\bar{y}^F,Z\right)$ be defined as follows:*

$$\Delta_t^D\left(y_j;\mu,n,\bar{y}^D,\bar{y}^F,Z\right)=\Pi_t^D\left(y_j;\mu,n,\bar{y}^D,\bar{y}^F,Z\right)-\Pi_t^{F,d}\left(y_j;\mu,n,\bar{y}^D,\bar{y}^F,Z\right), \tag{17}$$

$$\Delta_t^F\left(y_j;\mu,n,\bar{y}^D,\bar{y}^F,Z\right)=\Pi_t^{D,d}\left(y_j;\mu,n,\bar{y}^D,\bar{y}^F,Z\right)-\Pi_t^F\left(y_j;\mu,n,\bar{y}^D,\bar{y}^F,Z\right). \tag{18}$$

7.2 Preliminary lemmas

In this section, we introduce an auxiliary system of equations and show that the variables of interest in the equilibria of our game converge to the solutions of this system of equations.

7.2.1 The auxiliary system of equations

Fix (μ,n) with $\mu\in[0,1]$ and $n\in[0,1]$. Define $I_D(n,\mu)$ and $I_F(n,\mu)$ to be measurable subsets of the support of $H(\cdot)$ such that $\mu=\int_{y_j\in I_D(n,\mu)}dH(y_j)+\int_{y_j\in I_F(n,\mu)}dH(y_j)$ and $n=\frac{\int_{y_j\in I_D(n,\mu)}dH(y_j)}{\mu}$. Define $\bar{y}^D\equiv\frac{\int_{y_j\in I_D(n,\mu)}y_j dH(y_j)}{\int_{y_j\in I_D(n,\mu)}dH(y_j)}$, and $\bar{y}^F\equiv\frac{\int_{y_j\in I_F(n,\mu)}y_j dH(y_j)}{\int_{y_j\in I_F(n,\mu)}dH(y_j)}$. Let $P(N|n)\equiv\binom{T}{N}n^N(1-n)^{T-N}$. Define

$$X_t^D\left(y_j,N,y^D,\bar{y}^F\right)\equiv\frac{\gamma\left(\sigma_v^2+\frac{1}{T-N}\left(\sigma_{\alpha Ft}^2+\sigma_\varepsilon^2\right)\right)+\frac{1}{N}y_j+\frac{N-1}{N}\bar{y}^D-\bar{y}^F}{\gamma N\left(2\sigma_v^2+\frac{1}{T-N}\left(\sigma_{\alpha Ft}^2+\sigma_\varepsilon^2\right)+\frac{1}{N}\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)\right)}+\frac{N-1}{N}\frac{y_j-\bar{y}^D}{\gamma\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)}, \tag{19}$$

for $N>0$ and 0 otherwise; and

$$X_t^F\left(y_j,N,y^D,\bar{y}^F\right)\equiv\frac{\gamma\left(\sigma_v^2+\frac{1}{N}\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)\right)+\frac{T-N-1}{T-N}\bar{y}^F+\frac{1}{T-N}y_j-\bar{y}^D}{\gamma\left(T-N\right)\left(2\sigma_v^2+\frac{1}{T-N}\left(\sigma_{\alpha Ft}^2+\sigma_\varepsilon^2\right)+\frac{1}{N}\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)\right)}+\frac{T-N-1}{T-N}\frac{y_j-\bar{y}^F}{\gamma\left(\sigma_{\alpha Ft}^2+\sigma_\varepsilon^2\right)}, \tag{20}$$

for $N<T$ and 0 otherwise. Define also the following:

$$\Pi_t^D\left(y_j;\mu,n,\bar{y}^D,\bar{y}^F\right)\equiv\frac{1}{\mu}\sum_{N=0}^{T}P(N|n)\frac{N}{n}X_t^D\left(y_j,N,y^D,\bar{y}^F\right) \tag{21}$$

$$\Pi_t^F\left(y_j;\mu,n,\bar{y}^D,\bar{y}^F\right)\equiv\frac{1}{\mu}\sum_{N=0}^{T}P\left(N|n\right)\frac{T-N}{1-n}X_t^F\left(y_j,N,y^D,\bar{y}^F\right) \tag{22}$$

$$\Delta_t\left(y_j;\mu,n,\bar{y}^D,\bar{y}^F\right)\equiv\Pi_t^D\left(y_j;\mu,n,\bar{y}^D,\bar{y}^F\right)-\Pi_t^F\left(y_j;\mu,n,\bar{y}^D,\bar{y}^F\right) \tag{23}$$

$$Q_{Dt}\left(\mu,n,\bar{y}^D,\bar{y}^F\right)\equiv\sum_{N=0}^{T}P\left(N|n\right)NE_H\left[X_t^D\left(y_j,N,y^D,\bar{y}^F\right)|y_j\in I_D\left(n,\mu\right)\right] \tag{24}$$

$$Q_{Ft}\left(\mu,n,\bar{y}^D,\bar{y}^F\right)\equiv\sum_{N=0}^{T}P\left(N|n\right)\left(T-N\right)E_H\left[X_t^F\left(y_j,N,y^D,\bar{y}^F\right)|y_j\in I_F\left(n,\mu\right)\right] \tag{25}$$

Definition (2) *Let $\left\{\mu_*,n_*,\bar{y}_*^D,\bar{y}_*^F\right\}$ be a quadruple that satisfies the following inequalities:*

$$\sum_{t=1}^{2}\Delta_t\left(y_j;\mu_*,n_*,\bar{y}_*^D,\bar{y}_*^F\right)\begin{cases}\geq F_D-F_F & \text{if } y_j\in I_D\left(\mu_*,n_*\right)\\ \leq F_D-F_F & \text{if } y_j\in I_F\left(\mu_*,n_*\right)\end{cases}; \tag{26}$$

$$\sum_{t=1}^{2}\Pi_t^D\left(y_j;\mu_*,n_*,\bar{y}_*^D,\bar{y}_*^F\right)\geq F_D \text{ if } y_j\in I_D\left(\mu_*,n_*\right); \tag{27}$$

$$\sum_{t=1}^{2}\Pi_t^F\left(y_j;\mu_*,n_*,\bar{y}_*^D,\bar{y}_*^F\right)\geq F_F \text{ if } y_j\in I_F\left(\mu_*,n_*\right); \tag{28}$$

$$\sum_{t=1}^{2}\Pi_t^M\left(y_j;\mu_*,n_*,\bar{y}_*^D,\bar{y}_*^F\right)\leq F_M \text{ for } M\in\{D,F\} \text{ if } y_j\notin I_D\left(\mu_*,n_*\right)\cup I_F\left(\mu_*,n_*\right). \tag{29}$$

7.2.2 Convergence of the equilibrium variables to $\left\{\mu_*,n_*,\bar{y}_*^D,\bar{y}_*^F\right\}$

Lemma 1 *For any Z, let K_Z be a set of $(n,\mu)\in[0,1]^2$ such that $n\mu Z$ and $(1-n)\mu Z$ are integers. Then as $Z\to\infty$, for all corresponding $(n,\mu)\in K_Z$, we have:*

(a) $\lim_{Z\to\infty}ZP_D\left(N|\mu,n,Z\right)=P\left(N|n\right)\frac{N}{\mu n}$,
$\lim_{Z\to\infty}ZP_{D,d}\left(N|\mu,n,Z\right)=P\left(N|n\right)\frac{N}{\mu n}$,
$\lim_{Z\to\infty}ZP_{D,o}\left(N|\mu,n,Z\right)=P\left(N|n\right)\frac{N}{\mu n}$;

(b) $\lim_{Z\to\infty}ZP_F\left(N|\mu,n,Z\right)=P\left(N|n\right)\frac{T-N}{\mu(1-n)}$,
$\lim_{Z\to\infty}ZP_F,d\left(N|\mu,n,Z\right)=P\left(N|n\right)\frac{T-N}{\mu(1-n)}$,
$\lim_{Z\to\infty}ZP_{F,o}\left(N|\mu,n,Z\right)=P\left(N|n\right)\frac{T-N}{\mu(1-n)}$;

(c) $\lim_{Z\to\infty}P\left(N|\mu,n,Z\right)=P\left(N|n\right)N$.

Proof. We will prove only the first statement, since the proofs of the remaining statements are analogous. Fix some $(n,\mu) \in K_Z$. Note that

$$
\begin{aligned}
ZP_D(N|\mu,n,Z) &= Z\frac{\binom{\mu nZ-1}{N-1}\binom{\mu(1-n)Z}{T-N}}{\binom{\mu Z}{T}} \\
&= \frac{ZN\prod_{k=0}^{N-2}(\mu nZ-1-k)\prod_{k=0}^{T-N-1}(\mu(1-n)Z-k)}{\prod_{k=0}^{T-1}(\mu Z-k)}\frac{T!}{N!(T-N)!} \\
&= \frac{n\prod_{k=0}^{N-2}\left(n-\frac{1+k}{\mu Z}\right)\prod_{k=0}^{T-N-1}\left((1-n)-\frac{k}{\mu Z}\right)}{\prod_{k=0}^{T-1}\left(1-\frac{k}{\mu Z}\right)}\frac{\frac{N}{n\mu}T!}{N!(T-N)!}.
\end{aligned}
$$

The difference between the last expression and $\frac{T!}{N!(T-N)!}n^N(1-n)^{T-N}\frac{N}{n\mu}$ becomes arbitrarily small as Z increases, which completes the proof of the first inequality. ■

Lemma 2 *Fix $\bar{y}^D, \bar{y}^F$, and let K_Z be a set of $(n,\mu) \in [0,1]^2$ such that $n\mu Z$ and $(1-n)\mu Z$ are integers. Then for each $\varepsilon > 0$, there exists $\hat{Z}$ such that for all $Z > \hat{Z}$ and all corresponding $(n,\mu) \in K_Z$, the following inequalities hold for $M = D, F$:*

(a) $\left|\Pi_t^M(y_j;\mu,n,\bar{y}^D,\bar{y}^F,Z) - \Pi_t^M(y_j;\mu,n,\bar{y}^D,\bar{y}^F)\right| < \varepsilon;$

$\left|\Pi_t^{M,o}(y_j;\mu,n,\bar{y}^D,\bar{y}^F,Z) - \Pi_t^M(y_j;\mu,n,\bar{y}^D,\bar{y}^F)\right| < \varepsilon;$

(b) $\left|\Delta_t^M(y_j;\mu,n,\bar{y}^D,\bar{y}^F,Z) - \Delta_t(y_j;\mu,n,\bar{y}^D,\bar{y}^F,Z)\right| < \varepsilon;$

(c) $\left|\frac{\partial\Delta_t^M(y_j;\mu,n,\bar{y}^D,\bar{y}^F,Z)}{\partial y_j} - \frac{\partial\Delta_t(y_j;\mu,n,\bar{y}^D,\bar{y}^F)}{\partial y_j}\right| < \varepsilon;$

(d) $\left|Q_{Mt}(\mu,n,\bar{y}^D,\bar{y}^F,Z) - Q_{Mt}(\mu,n,\bar{y}^D,\bar{y}^F)\right| < \varepsilon.$

Proof. *Using the fact that $\lim_{Z\to\infty}\frac{n\mu Z\bar{y}^D-y_j}{n\mu Z-1} = \bar{y}^D$ and $\lim_{Z\to\infty}\frac{(1-n)\mu Z\bar{y}^F-y_j}{(1-n)\mu Z-1} = \bar{y}^F$ in (15) and (16), we establish that $\lim_{Z\to\infty}E\left[x_{jt}^M|y_j,N,\bar{y}^D,\bar{y}^F,Z\right] = X_t^M(y_j,N,\bar{y}^D,\bar{y}^F)$. Taking the derivative of (15) and (19) with respect to y_j, we obtain:*

$$
\lim_{Z\to\infty}\frac{\partial E\left[x_{jt}^D|y_j,N,\bar{y}^D,\bar{y}^F,Z\right]}{\partial y_j} = \frac{\frac{1}{N}}{\gamma N\left(2\sigma_v^2+\frac{1}{T-N}(\sigma_{\alpha Ft}^2+\sigma_\varepsilon^2)+\frac{1}{N}(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2)\right)} + \frac{N-1}{\gamma N(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2)} = \frac{\partial X_t^D(y_j,N,y^D,\bar{y}^F)}{\partial y_j}. \quad (30)
$$

An analogous analysis holds for $M = F$. Using this together with Lemma 1 in equations (21), (22), (2), and (14) completes the proof of part (a), and doing the same in equations (17), (18), and (23) completes the proof of parts (b) and (c).

For part (d), note that $E[y_j|y_j \in I_M(n,\mu)] = \bar{y}^M$. Using this in equations (19) and (20), we obtain that $E\left[X_t^M(y_j,N,y^D,\bar{y}^F)|y_j \in I_M(n,\mu)\right] = X_t^M(\bar{y}^M,N,y^D,\bar{y}^F)$. Plugging this *into (24) and (25) results in the following:*

$$Q_{Dt}\left(\mu,n,\bar{y}^D,\bar{y}^F\right)=n\mu\sum_{N=0}^{T}P\left(N|n\right)\frac{N}{n\mu}X_t^D\left(\bar{y}^D,N,y^D,\bar{y}^F\right)=n\mu\Pi_t^D\left(\bar{y}^D;\mu,n,\bar{y}^D,\bar{y}^F\right), \tag{31}$$

$$Q_{Ft}\left(\mu,n,\bar{y}^D,\bar{y}^F\right)=\left(1-n\right)\mu\Pi_t^F\left(\bar{y}^F;\mu,n,\bar{y}^D,\bar{y}^F\right). \tag{32}$$

Recall that $Q_{Mt}\left(\mu_Z,n_Z,\bar{y}_Z^D,\bar{y}_Z^F,Z\right)=E\left[q_t^M\left(N\right)|\mu_Z,n_Z,\bar{y}_Z^D,\bar{y}_Z^F,Z\right]$. *Taking the expectation of (10) conditional on* $\mu,n,\bar{y}^D,\bar{y}^F$, *and* Z, *and using the fact that* $E\left[\frac{1}{N}\sum_{i=1}^{N}\phi_{it}^D|\mu,n,\bar{y}^D,\bar{y}^F,Z\right]=\bar{y}^D$, *we obtain that* $E\left[q_t^M\left(N\right)|N,\bar{y}_Z^D,\bar{y}_Z^F,Z\right]=Nx_{jt}^D\left(N\right)$, *where* j *is a manager for whom* $y_j=\bar{y}^D$. *Moreover,* $P\left(N|\mu_Z,n_Z,Z\right)=ZP_D\left(N|\mu_Z,n_Z,Z\right)\frac{\mu nZ}{N}$. *Hence, we obtain:*

$$Q_{Mt}\left(\mu_Z,n_Z,\bar{y}_Z^D,\bar{y}_Z^F,Z\right)=\mu nZ\sum_{N=0}^{T}ZP_D\left(N|\mu_Z,n_Z,Z\right)x_{jt}^D\left(N\right)=n\mu\Pi_t^D\left(\bar{y}^D;\mu_Z,n_Z,\bar{y}_Z^D,\bar{y}_Z^F,Z\right), \tag{33}$$

Using a similar argument, we obtain:

$$Q_{Ft}\left(\mu_Z,n_Z,\bar{y}_Z^D,\bar{y}_Z^F,Z\right)=\left(1-n\right)\mu\Pi_t^F\left(\bar{y}^F;\mu_Z,n_Z,\bar{y}_Z^D,\bar{y}_Z^F,Z\right). \tag{34}$$

This, together with part (a) *implies part* (d). ■

The next lemma establishes that as Z increases, the distribution of signals $\{y_i\}_{i=1}^{Z}$ becomes arbitrarily close to $H(\cdot)$. Lemma 4 establishes that as Z increases, the properties of all equilibria converge to the properties of some of the solutions of the auxiliary set defined in Definition (1).

Lemma 3 *We have* $\lim_{Z\to\infty}\sup_s\left|\frac{1}{s}\sum_{i=1}^{Z}\mathbf{1}\left\{y_i\le s\right\}-H\left(s\right)\right|=0$ *with probability* 1.

Proof. For any $i\neq j$, y_i and y_j are *i.i.d.* with c.d.f. $H(\cdot)$. Hence, the lemma follows directly from the Glivenko–Cantelli theorem. ■

Lemma 4 *Let the quadruple* $\left\{\mu_Z,n_Z,\bar{y}_Z^D,\bar{y}_Z^F\right\}$ *correspond to some equilibrium for* Z. *For all* $\varepsilon>0$, *we can find* $\hat{Z}$ *and* $\left\{\mu_*,n_*,\bar{y}_*^D,\bar{y}_*^F\right\}$ *satisfying Definition (2) such that for all* $Z>\hat{Z}$, *we have* $\left|\left\{\mu_Z,n_Z,\bar{y}_Z^D,\bar{y}_Z^F\right\}-\left\{\mu_*,n_*,\bar{y}_*^D,\bar{y}_*^F\right\}\right|<\varepsilon$ *with probability* 1.

Proof. Suppose, by contradiction, that there exists a subsequence $\{Z_m\}$, for which $\lim_{Z_m\to\infty}\left\{\mu_{Z_m},n_{Z_m},\bar{y}_{Z_m}^D,\bar{y}_{Z_m}^F\right\}\neq\left\{\mu_*,n_*,\bar{y}_*^D,\bar{y}_*^F\right\}$. Because the set of possible values of $\mu_Z,n_Z,\bar{y}_Z^D,\bar{y}_Z^F$ is compact, there exists a subsequence $\{Z_k\}$ for which $\left\{\mu_{Z_k},n_{Z_k},\bar{y}_{Z_k}^D,\bar{y}_{Z_k}^F\right\}$ converges. Let $\left\{\mu_c,n_c,\bar{y}_c^D,\bar{y}_c^F\right\}$ be this limit. If $\left\{\mu_c,n_c,\bar{y}_c^D,\bar{y}_c^F\right\}\neq\left\{\mu_*,n_*,\bar{y}_*^D,\bar{y}_*^F\right\}$, then by definition of the latter quadruple, the former quadruple must violate at least one of conditions (26), (27), (28), and (29). That is, there exists a y_j for which at least one of the following must hold:

$$\begin{aligned}
[1] &: \sum_{t=1}^{2} \Delta_t \left(y_j; \mu_c, n_c, \bar{y}_c^D, \bar{y}_c^F\right) < F_D - F_F \text{ if } j \in I_D(n_c, \mu_c) \\
[2] &: \sum_{t=1}^{2} \Delta_t \left(y_j; \mu_c, n_c, \bar{y}_c^D, \bar{y}_c^F\right) > F_D - F_F \text{ if } j \in I_F(n_c, \mu_c) \\
[3] &: \sum_{t=1}^{2} \Pi_t^D \left(y_j; \mu_c, n_c, \bar{y}_c^D, \bar{y}_c^F\right) < F_D \text{ if } j \in I_D(n_c, \mu_c) \\
[4] &: \sum_{t=1}^{2} \Pi_t^F \left(y_j; \mu_c, n_c, \bar{y}_c^D, \bar{y}_c^F\right) < F_F \text{ if } j \in I_F(n_c, \mu_c) \\
[5] &: \sum_{t=1}^{2} \Pi_t^M \left(y_j; \mu_c, n_c, \bar{y}_c^D, \bar{y}_c^F\right) > F_M \text{ for } M \in \{D, F\} \text{ if } j \notin I_D(n_c, \mu_c) \cup I_F(n_c, \mu_c).
\end{aligned}$$

By parts (a) and (b) of Lemma 2, for Z large enough, one of the corresponding equilibrium conditions for Z must be violated for y_j. By the continuity of (2),(3), and (14) in y_j, there exists $\xi > 0$ such that the violated condition is also violated for all $y_i \in (y_j - \xi, y_j + \xi)$. It remains for us to show that for each Z large enough, one can find $y_i \in (y_j - \xi, y_j + \xi)$.

Suppose, by contradiction, that there exists a subsequence of economies $\{Z_k\}$ for which $y_i \notin (y_j - \xi, y_j + \xi)$ for all y_i and some $\xi > 0$. Then for all such Z_k, $\sum_{i=1}^{Z_k} \mathbf{1}\{y_i \leq y_j - \xi\} = \sum_{i=1}^{Z_k} \mathbf{1}\{y_i \leq y_j + \xi\}$. Note also, that the following holds:

$$\begin{aligned}
& \lim_{Z_k \to \infty} \sup_s \left| \frac{1}{s} \sum_{i=1}^{Z_k} \mathbf{1}\{y_i \leq s\} - H(s) \right| \\
\geq \quad & \max \left\{ \left| \frac{1}{y_j - \xi} \sum_{i=1}^{Z_k} \mathbf{1}\{y_i \leq y_j - \xi\} - H(y_j - \xi) \right|, \left| \frac{1}{y_j + \xi} \sum_{i=1}^{Z_k} \mathbf{1}\{y_i \leq y_j + \xi\} - H(y_j + \xi) \right| \right\}.
\end{aligned}$$

Suppose, without loss of generality, that the first expression on the right-hand side is 0, i.e., $\frac{1}{y_j - \xi} \sum_{i=1}^{Z_k} \mathbf{1}\{y_i \leq y_j - \xi\} - H(y_j - \xi) = 0$. Using this and the fact that $\sum_{i=1}^{Z_k} \mathbf{1}\{y_i \leq y_j - \xi\} = \sum_{i=1}^{Z_k} \mathbf{1}\{y_i \leq y_j + \xi\}$, we obtain that the second expression on the right-hand side is $\frac{y_j - \xi}{y_j + \xi} H(y_j - \xi) - H(y_j + \xi)$. Since $\frac{y_j - \xi}{y_j + \xi} < 1$, and by Assumption C, $H(y_j - \xi) < H(y_j + \xi)$, it follows that $\left| \frac{1}{y_j + \xi} \sum_{i=1}^{Z_k} \mathbf{1}\{y_i \leq y_j + \xi\} - H(y_j + \xi) \right| > 0$. But from Lemma 3, we know that the following is true $\lim_{Z_k \to \infty} \sup_s \left| \frac{1}{s} \sum_{i=1}^{Z_k} \mathbf{1}\{y_i \leq s\} - H(s) \right| = 0$, which is a contradiction. ■

7.3 Lemmas used later in the proofs of propositions 1, 2, and 3

In the subsequent proofs, to simplify notation we assume that T is odd, but the proofs hold when T is even as well. To simplify notation, let $\Delta(\cdot) \equiv \Delta_1(\cdot) + \Delta_2(\cdot)$ and $\Delta^M(\cdot) \equiv \Delta_1^M(\cdot) + \Delta_2^M(\cdot)$. And finally, we extensively use that fact that $\sigma^2_{\alpha Dt} \leq \sigma^2_{\alpha Ft}$.

Lemma 5 *We have*

$$\sum_{N=0}^{T} P(N|n) \frac{N\sigma_v^2 (1-2n)(T-N) + \left(\sigma_\varepsilon^2 + \sigma_{\alpha Dt}^2\right)(N - Tn)}{2N(T-N)\sigma_v^2 + T\left(\sigma_{\alpha Dt}^2 + \sigma_\varepsilon^2\right)} \begin{cases} > 0 \text{ for } n < \frac{1}{2} \\ < 0 \text{ for } n > \frac{1}{2} \end{cases}.$$

Proof. Taking the derivative of the left-hand side with respect to σ_v^2, one obtains:

$$\begin{aligned}
\frac{dLHS}{d\sigma_v^2} &= \sum_{N=0}^{T} P\left(N|n\right) \left(\frac{N\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)(T-N)(T-2N)}{\left(2N(T-N)\sigma_v^2+T\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)\right)^2} \right) \\
&= \sum_{N=0}^{T} P\left(N|n\right) \frac{N\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)(T-N)(T-2N)}{\left(2N(T-N)\sigma_v^2+T\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)\right)^2} \left(1-\left(\frac{n}{1-n}\right)^{T-2N}\right) \begin{cases} >0 \text{ if } n<\frac{1}{2} \\ =0 \text{ when } n=\frac{1}{2} \\ <0 \text{ if } n>\frac{1}{2} \end{cases} ,
\end{aligned}$$

where the second equality uses the fact that the expression in the bracket is the same for $N=k$ and $N=T-k$. Therefore, setting $\sigma_v^2=0$ yields

$$\sum_{N=0}^{T} P\left(N|n\right) \frac{N\sigma_v^2\left(1-2n\right)\left(T-N\right)+\left(\sigma_\varepsilon^2+\sigma_{\alpha Dt}^2\right)\left(N-Tn\right)}{2N\left(T-N\right)\sigma_v^2+T\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)} \begin{cases} >\sum_{N=0}^{T} P\left(N|n\right)\frac{N-Tn}{T}0 \text{ for } n<\frac{1}{2} \\ <\sum_{N=0}^{T} P\left(N|n\right)\frac{N-Tn}{T}=0 \text{ for } n>\frac{1}{2} \end{cases} .$$

■

If we interpret the auxiliary system of equations as a description of a limit economy, the next lemmas establish the following. Lemma 6 states that when there are fewer domestic-asset markets, and they are on average of lower quality, an average foreign-asset manager finds it profitable to deviate to domestic assets. Lemma 7 establishes that if the opposite is true and the updating about the managers' abilities is the same in both markets, then an average domestic-asset manager finds it profitable to deviate to foreign assets. Lemmas 8 and Lemma 9 state that if there are more domestic-asset managers, then the most able managers have the highest incentive to specialize domestically.

Lemma 6 *If $n\le\frac{1}{2}$ and $\bar{y}^D\le\bar{y}^F$, then $\Delta\left(\bar{y}^F;\mu,n,\bar{y}^D,\bar{y}^F\right)\ge 0$. The last inequality is strict if either $\bar{y}^D<\bar{y}^F$ or $\sigma_{\alpha F2}^2>\sigma_{\alpha D2}^2$.*

Proof. Plug (19) and (20) into (21) and (22), respectively, and plug the resulting equations into (23), to obtain

$$\begin{aligned}
&\mu\Delta_t\left(y_j;\mu,n,\bar{y}^D,\bar{y}^F\right)= \qquad (35)\\
=\quad &\sum_{N=1}^{T-1} P\left(N|n\right)\frac{N}{n}\left(\frac{\gamma\left((T-N)N\sigma_v^2+N\left(\sigma_{\alpha Ft}^2+\sigma_\varepsilon^2\right)\right)+(T-N)N\left(\frac{1}{N}y_j+\frac{N-1}{N}\bar{y}^D-\bar{y}^F\right)}{\gamma N\left(2(T-N)N\sigma_v^2+N\left(\sigma_{\alpha Ft}^2+\sigma_\varepsilon^2\right)+(T-N)\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)\right)}+\frac{N-1}{N}\frac{y_j-\bar{y}^D}{\gamma\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)}\right)\\
&-\sum_{N=1}^{T-1} P\left(N|n\right)\frac{T-N}{1-n}\left(\frac{\gamma\left((T-N)N\sigma_v^2+(T-N)\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)\right)+(T-N)N\left(\frac{T-N-1}{T-N}\bar{y}^F+\frac{1}{T-N}y_j-\bar{y}^D\right)}{\gamma(T-N)\left(2(T-N)N\sigma_v^2+N\left(\sigma_{\alpha Ft}^2+\sigma_\varepsilon^2\right)+(T-N)\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)\right)}+\frac{T-N-1}{T-N}\frac{y_j-\bar{y}^F}{\gamma\left(\sigma_{\alpha Ft}^2+\sigma_\varepsilon^2\right)}\right)\\
&+P\left(T|n\right)\frac{1}{n}\left(1+(T-1)\frac{y_j-\bar{y}^D}{\gamma\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)}\right)-P\left(0|n\right)\frac{1}{1-n}\left(1+(T-1)\frac{y_j-\bar{y}^F}{\gamma\left(\sigma_{\alpha Ft}^2+\sigma_\varepsilon^2\right)}\right).
\end{aligned}$$

Using $y_j=\bar{y}^F$, and grouping terms with $\bar{y}^D$ and $\bar{y}^F$, we obtain:

$$\begin{aligned}
&\mu\left(1-n\right)n\Delta_t\left(\bar{y}^F;\mu,n,\bar{y}^D,\bar{y}^F\right)=\\
=\quad &\sum_{N=1}^{T-1} P\left(N|n\right)\underbrace{\frac{\gamma\left(\sigma_v^2\left(1-2n\right)+\frac{(1-n)}{(T-N)}\left(\sigma_{\alpha Ft}^2+\sigma_\varepsilon^2\right)-\frac{n}{N}\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)\right)-\frac{(N+n-1)}{N}\left(\bar{y}^F-\bar{y}^D\right)}{\gamma\left(2\sigma_v^2+\frac{N}{N(T-N)}\left(\sigma_{\alpha Ft}^2+\sigma_\varepsilon^2\right)+\frac{(T-N)}{N(T-N)}\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)\right)}}_{\equiv FT}\\
&+\sum_{N=1}^{T-1} P\left(N|n\right)\frac{\left(\bar{y}^F-\bar{y}^D\right)}{\gamma\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)}\left(1-n\right)\left(N-1\right)+P\left(T|n\right)\left(1-n\right)\left(1+(T-1)\frac{y^F-\bar{y}^D}{\gamma\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)}\right)-nP\left(0|n\right) .
\end{aligned}$$

Differentiating FT with respect to $\sigma^2_{\alpha Ft}$, we establish that

$$\frac{\partial FT}{\partial\sigma^2_{\alpha Ft}} = \frac{N(T-N)\left(\gamma\sigma^2_{\alpha Dt}+\gamma\sigma^2_{\varepsilon}+(N+n-1)\left(\bar{y}^F-\bar{y}^D\right)+N\gamma\sigma^2_v\right)}{\gamma\left(2N(T-N)\sigma^2_v+N\left(\sigma^2_{\alpha Ft}+\sigma^2_{\varepsilon}\right)+(T-N)\left(\sigma^2_{\alpha Dt}+\sigma^2_{\varepsilon}\right)\right)^2} > 0.$$

Since $\sigma^2_{\alpha Ft}\geq\sigma^2_{\alpha Dt}$, the fact that $\frac{\partial FT}{\partial\sigma^2_{\alpha Ft}}>0$ implies that when we set $\sigma^2_{\alpha Ft}=\sigma^2_{\alpha Dt}$, we obtain:

$$\begin{aligned}
&\mu(1-n)n\Delta_t\left(\bar{y}^F;\mu,n,\bar{y}^D,\bar{y}^F\right) = \\
\geq\ &\textstyle\sum_{N=0}^{T}\frac{\gamma\left(N\sigma^2_v(1-2n)(T-N)+(N-Tn)\left(\sigma^2_{\alpha Dt}+\sigma^2_{\varepsilon}\right)\right)}{\gamma\left(2N(T-N)\sigma^2_v+T\left(\sigma^2_{\alpha Dt}+\sigma^2_{\varepsilon}\right)\right)}-\sum_{N=1}^{T-1}\frac{(N+n-1)(T-N)\left(\bar{y}^F-\bar{y}^D\right)}{\gamma\left(2N(T-N)\sigma^2_v+T\left(\sigma^2_{\alpha Dt}+\sigma^2_{\varepsilon}\right)\right)} \\
&\textstyle+\sum_{N=1}^{T}P(N|n)\frac{\left(\bar{y}^F-\bar{y}^D\right)}{\gamma\left(\sigma^2_{\alpha Dt}+\sigma^2_{\varepsilon}\right)}(1-n)(N-1)\ ,
\end{aligned}$$

with strict inequality for $t=2$ if $\sigma^2_{\alpha F2}>\sigma^2_{\alpha D2}$. Using Lemma 5 in the above equation, we obtain:

$$\begin{aligned}
\mu(1-n)n\Delta_t\left(\bar{y}^F;\mu,n,\bar{y}^D,\bar{y}^F\right) &\geq \frac{\left(\bar{y}^F-\bar{y}^D\right)}{\gamma}\sum_{N=1}^{T}P(N|n)\left(\frac{(1-n)(N-1)}{\left(\sigma^2_{\alpha Dt}+\sigma^2_{\varepsilon}\right)}-\frac{(N+n-1)(T-N)}{\left(2N(T-N)\sigma^2_v+T\left(\sigma^2_{\alpha Dt}+\sigma^2_{\varepsilon}\right)\right)}\right) \\
&> \frac{\left(\bar{y}^F-\bar{y}^D\right)}{\gamma}\sum_{N=0}^{T}P(N|n)N\frac{N+n-Tn-1}{T\gamma\left(\sigma^2_{\varepsilon}+\sigma^2_{\alpha Dt}\right)}=0\ ,
\end{aligned}$$

where the last inequality is strict if $\bar{y}^D<\bar{y}^F$, and the last equality comes from $E\left[N^2\right]=Tn-Tn^2+(Tn)^2$. ■

Lemma 7 *Suppose that* $\sigma^2_{\alpha Ft}=\sigma^2_{\alpha Dt}$ *for* $t=1,2$. *Then* $n>\frac{1}{2}$ *and* $\bar{y}^D>\bar{y}^F$ *imply that* $\Delta\left(\bar{y}^D;\mu,n,\bar{y}^D,\bar{y}^F\right)<0$.

Proof. Plugging $y_j=\bar{y}^D$ and $\sigma^2_{\alpha Ft}=\sigma^2_{\alpha Dt}$ into (35), we obtain:

$$\begin{aligned}
\Delta_t\left(\bar{y}^D;\mu,n,\bar{y}^D,\bar{y}^F\right) &= \frac{1}{\mu(1-n)n}\sum_{N=1}^{T-1}P(N|n)\frac{N\sigma^2_v(1-2n)(T-N)+(N-Tn)\left(\sigma^2_{\alpha Dt}+\sigma^2_{\varepsilon}\right)}{2N(T-N)\sigma^2_v+T\left(\sigma^2_{\alpha Dt}+\sigma^2_{\varepsilon}\right)} \\
&\quad+\frac{\left(\bar{y}^D-\bar{y}^F\right)}{\mu(1-n)n}\sum_{N=1}^{T-1}P(N|n)\left(\frac{N(T-N-n)}{\gamma\left(2N(T-N)\sigma^2_v+T\left(\sigma^2_{\alpha Dt}+\sigma^2_{\varepsilon}\right)\right)}-\frac{(T-N-1)n}{\gamma\left(\sigma^2_{\alpha Dt}+\sigma^2_{\varepsilon}\right)}\right) \\
&\quad+P(T|n)\frac{1}{\mu n}-P(0|n)\frac{1}{\mu(1-n)}\left(1+(T-1)\frac{\bar{y}^D-\bar{y}^F}{\gamma\left(\sigma^2_{\alpha Dt}+\sigma^2_{\varepsilon}\right)}\right) \\
&= \frac{1}{\mu(1-n)n}\sum_{N=0}^{T}P(N|n)\frac{N\sigma^2_v(1-2n)(T-N)+(N-Tn)\left(\sigma^2_{\alpha Dt}+\sigma^2_{\varepsilon}\right)}{2N(T-N)\sigma^2_v+T\left(\sigma^2_{\alpha Dt}+\sigma^2_{\varepsilon}\right)} \\
&\quad+\frac{\bar{y}^D-\bar{y}^F}{\mu(1-n)n}\sum_{N=0}^{T-1}P(N|n)\left(\frac{N(T-N-n)}{\gamma\left(2N(T-N)\sigma^2_v+T\left(\sigma^2_{\alpha Dt}+\sigma^2_{\varepsilon}\right)\right)}-\frac{(T-N-1)n}{\gamma\left(\sigma^2_{\alpha Dt}+\sigma^2_{\varepsilon}\right)}\right)\ .
\end{aligned}$$

Using Lemma 5, for $n>\frac{1}{2}$ we have:

$$\begin{aligned}
\Delta_t\left(\bar{y}^D;\mu,n,\bar{y}^D,\bar{y}^F\right) &< \frac{\bar{y}^D-\bar{y}^F}{\mu(1-n)n}\sum_{N=0}^{T-1}P(N|n)\left(\frac{N(T-N-n)}{\gamma\left(2N(T-N)\sigma^2_v+T\left(\sigma^2_{\alpha Dt}+\sigma^2_{\varepsilon}\right)\right)}-\frac{(T-N-1)n}{\gamma\left(\sigma^2_{\alpha Dt}+\sigma^2_{\varepsilon}\right)}\right) \\
&< \frac{\bar{y}^D-\bar{y}^F}{\mu(1-n)n}\sum_{N=0}^{T-1}P(N|n)\left(\frac{N(T-N-n)}{\gamma T\left(\sigma^2_{\alpha Dt}+\sigma^2_{\varepsilon}\right)}-\frac{(T-N-1)n}{\gamma\left(\sigma^2_{\alpha Dt}+\sigma^2_{\varepsilon}\right)}\right) \\
&= \frac{\left(\bar{y}^F-\bar{y}^D\right)}{\gamma}\frac{1}{\mu(1-n)n}\sum_{N=0}^{T}P(N|n)\frac{(T-N)(N+n-Tn)}{\left(T\left(\sigma^2_{\alpha Dt}+\sigma^2_{\varepsilon}\right)\right)}=0\ ,
\end{aligned}$$

where the last equality comes from the fact that $E\left[N^2\right]=Tn-Tn^2+(Tn)^2$. ■

Lemma 8 *Suppose that $\sigma^2_{\alpha Ft} = \sigma^2_{\alpha Dt}$. Then, for $t = 1, 2$ and all y_j, we have*

$$\frac{\partial \Delta_t \left(y_j; \mu, n, \bar{y}^D, \bar{y}^F\right)}{\partial y_j} = \begin{cases} < 0 \text{ if } n < \frac{1}{2} \\ = 0 \text{ if } n = \frac{1}{2} \\ > 0 \text{ if } n > \frac{1}{2} \end{cases} . \tag{36}$$

Proof. Differentiating (35) with respect to y_j, we obtain:

$$\begin{aligned} \frac{\partial \Delta_t\left(y_j; n, \bar{y}^D, \bar{y}^F\right)}{\partial y_j} &= \frac{1}{\mu(1-n)n} \sum_{N=1}^{T-1} P\left(N|n\right) \frac{T(1-n)-N}{\gamma\left(2N(T-N)\sigma_v^2 + N\left(\sigma^2_{\alpha Ft}+\sigma^2_\varepsilon\right) + (T-N)\left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)\right)} \\ &+ \frac{1}{\mu(1-n)n} \sum_{N=1}^{T-1} P\left(N|n\right) \left(\frac{(N-1)(1-n)}{\gamma\left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)} - \frac{(T-N-1)n}{\gamma\left(\sigma^2_{\alpha Ft}+\sigma^2_\varepsilon\right)}\right) + P\left(T|n\right) \frac{1}{n\mu} \frac{T-1}{\gamma\left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)} - P\left(0|n\right) \frac{T-1}{(1-n)\mu} \frac{1}{\gamma\left(\sigma^2_{\alpha Ft}+\sigma^2_\varepsilon\right)} . \end{aligned} \tag{37}$$

Substituting $\sigma^2_{\alpha Dt}$ for $\sigma^2_{\alpha Ft}$ in the above expression, and using the facts that $\sigma^2_{\alpha Dt}$ is independent of N and that $\sum_{N=0}^{T} P\left(N|n\right) N = nT$, we obtain:

$$\begin{aligned} \frac{\partial \Delta_t\left(y_j; n, \bar{y}^D, \bar{y}^F\right)}{\partial y_j} &= \frac{1}{\mu(1-n)n} \sum_{N=1}^{T-1} P\left(N|n\right) \frac{T(1-n)-N}{\gamma\left(2N(T-N)\sigma_v^2 + T\left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)\right)} \\ &+ \frac{1}{\mu(1-n)n} \sum_{N=1}^{T-1} P\left(N|n\right) \left(\frac{(N-1)(1-n)}{\gamma\left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)} - \frac{(T-N-1)n}{\left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)}\right) + P\left(T|n\right) \frac{1}{n\mu} \frac{T-1}{\gamma\left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)} - P\left(0|n\right) \frac{T-1}{(1-n)\mu} \frac{1}{\gamma\left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)} . \end{aligned} \tag{38}$$

Rearranging terms, we obtain:

$$\begin{aligned} \frac{\partial \Delta_t\left(y_j; \mu, n, \bar{y}^D, \bar{y}^F\right)}{\partial y_j} &= \frac{\left(\sum_{N=1}^{T-1} P\left(N|n\right) \frac{2(2n-1)N(T-N)\sigma_v^2 - (N-Tn)\left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)}{\left(2N(T-N)\sigma_v^2 + T\left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)\right)} + \left(P\left(T|n\right)(1-n) - P\left(0|n\right)n\right)\right)}{\mu n (1-n) \gamma \left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)} = \\ &= \frac{\left(\sum_{N=0}^{T} P\left(N|n\right) \frac{2(2n-1)N(T-N)\sigma_v^2 - (N-Tn)\left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)}{\left(2N(T-N)\sigma_v^2 + T\left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)\right)} + 2n^T(1-n)\left(1 - \left(\frac{1-n}{n}\right)^{T-1}\right)\right)}{\mu n (1-n) \gamma \left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)}, \end{aligned}$$

which by Lemma 5 implies (36). ■

Lemma 9 *Suppose $\sigma^2_{\alpha F2} > \sigma^2_{\alpha D2}$. Then $n \geq \frac{1}{2}$ implies $\frac{\partial \Delta\left(y_j; \mu, n, \bar{y}^D, \bar{y}^F\right)}{\partial y_j} > 0$.*

Proof. Differentiating the right-hand side of (37) with respect to $\sigma^2_{\alpha Ft}$ yields

$$\begin{aligned} \frac{\partial^2 \Delta_t\left(y_j; n, \bar{y}^D, \bar{y}^F\right)}{\partial y_j \partial \sigma^2_{\alpha Ft}} &= \frac{1}{\mu(1-n)n} \sum_{N=1}^{T-1} P\left(N|n\right) \frac{(N-T(1-n))}{\gamma\left(2(T-N)\sigma_v^2 + \left(\sigma^2_{\alpha Ft}+\sigma^2_\varepsilon\right) + \frac{(T-N)}{N}\left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)\right)^2} \\ &+ \frac{1}{\mu(1-n)n} \sum_{N=1}^{T-1} P\left(N|n\right) \left(\frac{(T-N-1)n}{\gamma\left(\sigma^2_{\alpha Ft}+\sigma^2_\varepsilon\right)^2}\right) + P\left(0|n\right) \frac{T-1}{(1-n)\mu} \frac{1}{\gamma\left(\sigma^2_{\alpha Ft}+\sigma^2_\varepsilon\right)^2} . \end{aligned}$$

The last expression is positive. Using the fact that

$$\left(2(T-N)\sigma_v^2 + \left(\sigma^2_{\alpha Ft}+\sigma^2_\varepsilon\right) + \frac{(T-N)}{N}\left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)\right)^2 > \left(\sigma^2_{\alpha Ft}+\sigma^2_\varepsilon\right)^2,$$

we obtain:

$$\frac{\partial^2 \Delta_t\left(y_j; n, \bar{y}^D, \bar{y}^F\right)}{\partial y_j \partial \sigma_{\alpha Ft}} > \frac{1}{\mu(1-n)n} \sum_{N=1}^{T-1} P\left(N|n\right) \frac{N(1-n) - n + T(2n-1)}{\gamma\left(2(T-N)\sigma_v^2 + \left(\sigma^2_{\alpha Ft}+\sigma^2_\varepsilon\right) + \frac{(T-N)}{N}\left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)\right)^2} > 0 .$$

This implies that since $\sigma^2_{\alpha F1} = \sigma^2_{\alpha D1}$ and $\sigma^2_{\alpha F2} > \sigma^2_{\alpha D2}$, the derivative $\frac{\partial \Delta_t\left(y_j;n,\bar{y}^D,\bar{y}^F\right)}{\partial y_j}$ is larger (for $t=1$) or strictly larger (for $t=2$) than the derivative $\frac{\partial \Delta_t\left(y_j;n,\bar{y}^D,\bar{y}^F\right)}{\partial y_j}$ evaluated at $\sigma^2_{\alpha Ft} = \sigma^2_{\alpha Dt}$. Since Lemma 8 showed the latter to be weakly greater than 0 for $n \geq \frac{1}{2}$, we established that $\frac{\partial \Delta\left(y_j;n,\bar{y}^D,\bar{y}^F\right)}{\partial y_j} > 0$. ■

7.4 Proof of Proposition 1

It is easy to show that the solution to investors' problems exists. By incorporating the investors' best responses into managers' profit functions, we can convert the game with given Z into a finite game. The existence of an equilibrium follows then directly from Nash (1951).

By Lemma 4, we only need to prove that for all $\left\{\mu_*, n_*, \bar{y}^D_*, \bar{y}^F_*\right\}$ defined in Definition (2) above, we have $n_* = \frac{1}{2}$ (Part i), $\bar{y}^D_* = \bar{y}^F_*$ (Part ii), and $Q_{Dt}\left(\mu_*, n_*, \bar{y}^D_*, \bar{y}^F_*\right) = Q_{Ft}\left(\mu_*, n_*, \bar{y}^D_*, \bar{y}^F_*\right)$ (Part iii). Note that since $\sigma^2_\eta = 0$, we have that $\sigma^2_{\alpha Ft} = \sigma^2_{\alpha Dt} = 0$ for $t = 1, 2$. Note also that $F_D - F_F = 0$.

Part (i)

Suppose, by contradiction, that $n_* \neq \frac{1}{2}$. Suppose further that $n_* < \frac{1}{2}$. By Lemma 8, $n_* < \frac{1}{2}$ implies $\frac{\partial \Delta\left(y_j;\mu_*,n_*,\bar{y}^D_*,\bar{y}^F_*\right)}{\partial y_j} < 0$. This means that there exists a threshold $\hat{y}$, such that $\Delta\left(y_j;\mu_*,n_*,\bar{y}^D_*,\bar{y}^F_*\right) < 0$ for $y_j > \hat{y}$ and $\Delta\left(y_j;\mu_*,n_*,\bar{y}^D_*,\bar{y}^F_*\right) > 0$ for $y_j < \hat{y}$. This, together with the fact that $H\left(y_j\right)$ is continuous (no mass points) and with condition (26), implies that $\bar{y}^D_* < \bar{y}^F_*$. However, $n_* < \frac{1}{2}$ and $\bar{y}^D_* < \bar{y}^F_*$ cannot satisfy Definition (2). By Lemma 6, if $n_* \leq \frac{1}{2}$, then $\Delta\left(\bar{y}^F_*;\mu_*,n_*,\bar{y}^D_*,\bar{y}^F_*\right) > 0$, which violates condition (26) of Definition (2). The case with $n > \frac{1}{2}$ is proven analogously, with the use of Lemma 7 for the last part.

Part (ii).

First, note that for $n = \frac{1}{2}$, using Lemmas 5 and 8 in equation (35), we obtain

$$
\begin{aligned}
\Delta_t\left(y_j;\mu,\tfrac{1}{2},\bar{y}^D,\bar{y}^F\right) \quad = \quad & \tfrac{2}{\mu}\textstyle\sum_{N=1}^{T-1} P\left(N|\tfrac{1}{2}\right)\left(\frac{\gamma(2N-T)\left(\sigma^2_{\alpha Dt}+\sigma^2_{\varepsilon}\right)+N(T-N)\left(\frac{2N-1}{N}\bar{y}^D-\frac{2(T-N)-1}{T-N}\bar{y}^F\right)}{\gamma\left(2N(T-N)\sigma^2_v+T\left(\sigma^2_{\alpha Dt}+\sigma^2_{\varepsilon}\right)\right)}+\frac{(T-N-1)\bar{y}^F-(N-1)\bar{y}^D}{\gamma\left(\sigma^2_{\alpha Dt}+\sigma^2_{\varepsilon}\right)}\right) \\
& + \tfrac{1}{\mu}2P\left(T|\tfrac{1}{2}\right)\left(1+(T-1)\frac{-\bar{y}^D}{\gamma\left(\sigma^2_{\alpha Dt}+\sigma^2_{\varepsilon}\right)}\right) - \tfrac{1}{\mu}2P\left(0|\tfrac{1}{2}\right)\left(1+(T-1)\frac{-\bar{y}^F}{\gamma\left(\sigma^2_{\alpha Dt}+\sigma^2_{\varepsilon}\right)}\right). \\
= \quad & 2\frac{\bar{y}^F-\bar{y}^D}{\mu\gamma}\underbrace{\left(\sum_{N=1}^{\frac{T-1}{2}} P\left(N|\frac{1}{2}\right)\left(\frac{T-4NT+4N^2}{2N\left(T-N\right)\sigma^2_v+T\left(\sigma^2_{\alpha Dt}+\sigma^2_{\varepsilon}\right)}+\frac{T-2}{\sigma^2_{\alpha Dt}+\sigma^2_{\varepsilon}}\right)+\frac{P\left(T|\frac{1}{2}\right)(T-1)}{\sigma^2_{\alpha Dt}+\sigma^2_{\varepsilon}}\right)}_{\equiv\Psi}
\end{aligned}
$$

where the last equality uses the fact that $P\left(N|\frac{1}{2}\right) = P\left(T-N|\frac{1}{2}\right)$. Since $T - 4NT + 4N^2 > 0$ for $N < \frac{T}{2}$, it follows that $\Psi > 0$. Suppose now, by contradiction, that $\bar{y}^D_* \neq \bar{y}^F_*$. This means that $\Delta_t\left(y_j;\mu_*,\frac{1}{2},\bar{y}^D_*,\bar{y}^F_*\right) = \left(\bar{y}^F_* - \bar{y}^D_*\right)\Psi \neq 0$; hence, it follows that $\Delta_t\left(y_j;\mu_*,\frac{1}{2},\bar{y}^D_*,\bar{y}^F_*\right)$ is either strictly positive for all y_j, or strictly negative for all y_j. This, in turn, implies that condition (26) is violated for either $j \in I_D\left(n,\mu\right)$ or $j \in I_F\left(n,\mu\right)$.

Part (iii).

Using (31) and (32), we obtain:

$$\begin{aligned}Q_{Dt}\left(\mu_*,\frac{1}{2},\bar{y}_*^D,\bar{y}_*^F\right)-Q_{Ft}\left(\mu_*,\frac{1}{2},\bar{y}_*^D,\bar{y}_*^F\right) &= \frac{1}{2}\mu\Pi_t^D\left(\bar{y}_*^D;\mu_*,\frac{1}{2},\bar{y}_*^D,\bar{y}_*^F\right)-\frac{1}{2}\mu\Pi_t^F\left(\bar{y}_*^F;\mu_*,\frac{1}{2},\bar{y}_*^D,\bar{y}_*^F\right)\\ &= \frac{1}{2}\mu_*\Delta_t\left(\bar{y}_*^F;\mu_*,\frac{1}{2},\bar{y}_*^D,\bar{y}_*^F\right).\end{aligned}$$

Since $n_*=\frac{1}{2}$ and $\sigma^2_{\alpha Dt}=\sigma^2_{\alpha Ft}$, Lemma 8 implies that $\Delta_t\left(y_j;\mu,\frac{1}{2},\bar{y}_*^D,\bar{y}_*^F\right)$ is independent of y_j. Hence, to satisfy Definition (2), it must be the case that $\Delta_t\left(y_j;\mu,\frac{1}{2},\bar{y}_*^D,\bar{y}_*^F\right)=0$, which, applied to the above equation, completes the proof.

7.5 Proof of Proposition 2

The existence of the equilibria for each Z follows from the same argument as the one presented in the proof of Proposition 1.

Using equation (24) and (25), and using $n=\frac{1}{2}$ and $\bar{y}^D=\bar{y}^F$, we obtain:

$$\begin{aligned}&Q_{Dt}\left(\mu,\frac{1}{2},\bar{y}^D,\bar{y}^F\right)-Q_{Ft}\left(\mu,\frac{1}{2},\bar{y}^D,\bar{y}^F\right)\\ =\;&\sum_{N=0}^{T}P\left(N|\frac{1}{2}\right)\left(\frac{\frac{1}{T-N}\left(\sigma^2_{\alpha Ft}+\sigma^2_\varepsilon\right)-\frac{1}{N}\left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)}{\gamma\left(2\sigma_v^2+\frac{1}{T-N}\left(\sigma^2_{\alpha Ft}+\sigma^2_\varepsilon\right)+\frac{1}{N}\left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)\right)}\right).\end{aligned}$$

The right-hand side of the above expression increases in $\sigma^2_{\alpha Ft}$, so when replacing $\sigma^2_{\alpha Ft}$ in this expression with $\sigma^2_{\alpha Dt}$, we obtain:

$$\begin{aligned}&Q_{Dt}\left(\mu,\frac{1}{2},\bar{y}^D,\bar{y}^F\right)-Q_{Ft}\left(\mu,\frac{1}{2},\bar{y}^D,\bar{y}^F\right)\\ \geq\;&\sum_{N=0}^{T}P\left(N|\frac{1}{2}\right)\frac{(2N-T)\left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)}{\gamma\left(2N\left(T-N\right)\sigma_v^2+T\left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)\right)},\end{aligned}$$

with equality for $t=1$ (since $\sigma^2_{\alpha F1}=\sigma^2_{\alpha D1}$) and strict inequality for $t=2$ (since $\sigma^2_{\alpha F2}>\sigma^2_{\alpha D2}$). Denote the expression under the summation by $\Psi(N)$. Since $P\left(N|\frac{1}{2}\right)=P\left(T-N|\frac{1}{2}\right)$, and $\Psi(N)=-\Psi(T-N)$, the last summation is therefore equal to 0, which completes the proof.

7.6 Proof of Proposition 3

By Lemma 4, we only need to prove that for all $\left\{\mu_*,n_*,\bar{y}_*^D,\bar{y}_*^F\right\}$ satisfying Definition (2), we have $\bar{y}_*^D>\bar{y}_*^F$ (Part i), we have $\frac{\sum_{t=1}^2 Q_{Dt}\left(\mu_*,n_*,\bar{y}_*^D,\bar{y}_*^F\right)}{n_*}>\frac{\sum_{t=1}^2 Q_{Ft}\left(\mu_*,n_*,\bar{y}_*^D,\bar{y}_*^F\right)}{1-n_*}$ (Part ii), and there exists $H(y_j)$ such that $n_*<\frac{1}{2}$ and $Q_{Dt}\left(\mu_*,n_*,\bar{y}_*^D,\bar{y}_*^F\right)>Q_{Ft}\left(\mu_*,n_*,\bar{y}_*^D,\bar{y}_*^F\right)$ (Part iii).

Proof of Part (i).

Suppose, by contradiction, that $\bar{y}_*^D\leq\bar{y}_*^F$. Suppose next that at the same time $n_*\leq\frac{1}{2}$. Since $\sigma^2_{\alpha F2}>\sigma^2_{\alpha D2}$, Lemma 6 implies that $\Delta\left(\bar{y}_*^F;\mu_*,n_*,\bar{y}_*^D,\bar{y}_*^F\right)>0$. Since $\Delta\left(y_j;\mu_*,n_*,\bar{y}_*^D,\bar{y}_*^F\right)$ is linear in y_j, we have that $\Delta\left(y_j;\mu_*,n_*,\bar{y}_*^D,\bar{y}_*^F\right)>0$ for either all $y_j\leq\bar{y}^F$ or all $y_j\geq\bar{y}^F$.

This means that $\Delta\left(y_j;\mu_*,n_*,\bar{y}_*^D,\bar{y}_*^F\right) > 0$ for some $y_j \in I_F\left(n_*,\mu_*\right)$. This in turn implies that condition (26) is violated for some $y_j \in I_F\left(\mu,n\right)$. Suppose then that $n_* > \frac{1}{2}$ instead. By Lemma 9, $\frac{\partial\Delta\left(y_j;\mu_*,n_*,\bar{y}_*^D,\bar{y}_*^F\right)}{\partial y_j} > 0$; hence, condition (26) says that the managers with the highest y_j belong to $I_D\left(n_*,\mu_*\right)$, which implies that $\bar{y}_*^D > \bar{y}_*^F$; which contradicts the starting hypothesis.

Proof of Part (ii).

Using (31) and (32), we obtain:

$$Q_{Dt}\left(\mu,n,\bar{y}^D,\bar{y}^F\right) = n\mu\Pi_t^D\left(\bar{y}^D;\mu,n,\bar{y}^D,\bar{y}^F\right), \tag{39}$$

$$Q_{Ft}\left(\mu,n,\bar{y}^D,\bar{y}^F\right) = \left(1-n\right)\mu\Pi_t^F\left(\bar{y}^F;\mu,n,\bar{y}^D,\bar{y}^F\right). \tag{40}$$

Using equations (24) and (25), we next obtain:

$$\begin{aligned}
& \frac{\sum_{t=1}^2 Q_{Dt}\left(\mu_*,n_*,\bar{y}_*^D,\bar{y}_*^F\right)}{n_Z} - \frac{\sum_{t=1}^2 Q_{Ft}\left(\mu_*,n_*,\bar{y}_*^D,\bar{y}_*^F\right)}{1-n_Z} \\
= \; & \mu\left(\sum_{t=1}^2 \Pi_t^D\left(\bar{y}^D;\mu,n,\bar{y}^D,\bar{y}^F\right) - \Pi_t^F\left(\bar{y}^F;\mu,n,\bar{y}^D,\bar{y}^F\right)\right) \\
> \; & \mu\Delta\left(\bar{y}^D;\mu_*,n_*,\bar{y}_*^D,\bar{y}_*^F\right),
\end{aligned}$$

where the last inequality comes from the fact that Π_t^F increases in y_j. By definition of $\bar{y}_*^D$, the set $I_D\left(\mu_*,z_*\right)$ must contain some y_j greater than and some y_j smaller than $\bar{y}_*^D$, and condition (26) requires that for these, $\Delta\left(y_j;\mu_*,n_*,\bar{y}_*^D,\bar{y}_*^F\right) \geq 0$. By the linearity of Δ in y_j, we have $\Delta\left(\bar{y}^D;\mu_*,n_*,\bar{y}_*^D,\bar{y}_*^F\right) \geq 0$, which completes the proof.

Proof of Part (iii).

For any quint $\left\{\mu,n,\bar{y}^D,\bar{y}^F,y^L\right\}$ with $\bar{y}^D > \bar{y}^F > y_L$, define

$$h\left(y_j\right) = \begin{cases} \frac{(1-n)\mu}{2c} & \text{if} \quad y_j \in \left[\bar{y}^F - c, \bar{y}^F + c\right] \\ \frac{n\mu}{2c} & \text{if} \quad y_j \in \left[\bar{y}^D - c, \bar{y}^D + c\right] \\ \frac{1-\mu}{2c} & \text{if} \quad y_j \in \left[y^L - c, y^L + c\right] \end{cases}, \tag{41}$$

where $c > 0$ is small enough so that $y^L + c < \bar{y}^F - c$ and $\bar{y}^F + c < \bar{y}^D - c$. The corresponding $H\left(\cdot\right)$ satisfies Assumption C.

First, we will show that there exists some $n' < \frac{1}{2}$, such that for all $n \in \left(n',\frac{1}{2}\right)$, if $\bar{y}^D > \bar{y}^F$, then $Q_{Dt}\left(\mu,n,\bar{y}^D,\bar{y}^F\right) > \frac{1}{2}$ for $t = 1,2$. For this purpose, note that using (24) we obtain:

$$\begin{aligned}
Q_{Dt}\left(\mu,n,\bar{y}^D,\bar{y}^F\right) - \frac{1}{2} \; &= \; \sum_{N=0}^{T} P\left(N|n\right) N E\left[x_{jt}^D|\bar{y}^D,N,\bar{y}^D,\bar{y}^F\right] - \frac{1}{2} = \\
&\geq \; \frac{1}{2}\sum_{N=1}^{T} P\left(N|n\right) \frac{N\left(\sigma_{Ft}^2+\sigma_\varepsilon^2\right) - \left(T-N\right)\left(\sigma_{Dt}^2+\sigma_\varepsilon^2\right)}{2\left(T-N\right)N\sigma_v^2 + N\left(\sigma_{Ft}^2+\sigma_\varepsilon^2\right) + \left(T-N\right)\left(\sigma_{Dt}^2+\sigma_\varepsilon^2\right)} \\
&\quad + \sum_{N=1}^{T} P\left(N|n\right) \frac{\left(T-N\right)N\left(\bar{y}^D-\bar{y}^F\right)}{2\left(T-N\right)N\sigma_v^2 + N\left(\sigma_{Ft}^2+\sigma_\varepsilon^2\right) + \left(T-N\right)\left(\sigma_{Dt}^2+\sigma_\varepsilon^2\right)}.
\end{aligned}$$

Since $\bar{y}^D > \bar{y}^F$, the second expression is strictly positive for $n \in (0,1)$, and the first expression is increasing in σ^2_{Ft}. Therefore, for $t \in \{1,2\}$, we have

$$\begin{aligned} Q_{Dt}\left(\mu, n, \bar{y}^D, \bar{y}^F\right) - \frac{1}{2} &> \frac{1}{2}\sum_{N=1}^{T} P(N|n)\left(\frac{\left(\sigma^2_\varepsilon + \sigma^2_{Dt}\right)(2N-T)}{2(T-N)N\sigma^2_v + T\left(\sigma^2_{Dt}+\sigma^2_\varepsilon\right)}\right) = \\ &= \frac{1}{2}\sum_{N=1}^{\frac{T-1}{2}} P(N|n)\left(\frac{\left(\sigma^2_\varepsilon + \sigma^2_{Dt}\right)(T-2N)}{2(T-N)N\sigma^2_v + T\left(\sigma^2_{Dt}+\sigma^2_\varepsilon\right)}\left(\left(\frac{n}{1-n}\right)^{T-2N} - 1\right)\right) + \frac{1}{2}P(T|n), \end{aligned}$$

and the last expression is strictly positive for $n = \frac{1}{2}$. Hence, by the continuity of $Q_{Dt}\left(\mu, n, \bar{y}^D, \bar{y}^F\right)$ in n, it follows that exists some $n' < \frac{1}{2}$ such that $Q_{Dt}\left(\mu, n, \bar{y}^D, \bar{y}^F\right) - \frac{1}{2} > 0$ for all $n \in \left(n', \frac{1}{2}\right)$.

Let us now propose a limit equilibrium in which $y_j \in I_M(\mu, n)$ if and only if $y_j \in \left[\bar{y}^M - c, \bar{y}^M + c\right]$, and $y_j \in \left[y^L - c, y^L + c\right]$ do not open mutual funds. We will now show that there exists an open set of quints $\left\{\mu, n, \bar{y}^D, \bar{y}^F, y^L\right\}$ with $n \in \left(n', \frac{1}{2}\right)$, and the corresponding open set of density functions $h(y_j)$, such that conditions (26), (27), (28), and (29) are satisfied for the proposed equilibrium. Note that by construction, in the proposed equilibrium the fraction of all managers that open a mutual fund is μ, and the fraction of operating managers that specialize in domestic assets is n. The average quality of managers specializing in assets M is $\bar{y}^M$.

To check condition (26), note that using equation (35), we have:

$$\begin{aligned} &\mu\Delta_t\left(\bar{y}^D; \mu, n, \bar{y}^D, \bar{y}^F\right) = \\ &= \Psi + \frac{\left(\bar{y}^D - \bar{y}^F\right)}{(1-n)n}\sum_{N=0}^{T-1} P(N|n)\left(\frac{N(T-N-n)}{\gamma\left(2(T-N)N\sigma^2_v + N\left(\sigma^2_{\alpha Ft}+\sigma^2_\varepsilon\right) + (T-N)\left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)\right)} - \frac{(T-N-1)n}{\gamma\left(\sigma^2_{\alpha Ft}+\sigma^2_\varepsilon\right)}\right), \end{aligned}$$

and

$$\begin{aligned} &\mu\Delta_t\left(\bar{y}^F; \mu, n, \bar{y}^D, \bar{y}^F\right) = \\ &= \Psi + \frac{\left(\bar{y}^D - \bar{y}^F\right)}{(1-n)n}\sum_{N=1}^{T} P(N|n)\left(\frac{(N+n-1)(T-N)}{\gamma\left(2(T-N)N\sigma^2_v + N\left(\sigma^2_{\alpha Ft}+\sigma^2_\varepsilon\right) + (T-N)\left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)\right)} - \frac{(N-1)(1-n)}{\gamma\left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)}\right), \end{aligned} \tag{42}$$

where

$$\Psi \equiv \frac{1}{(1-n)n}\sum_{N=0}^{T} P(N|n)\frac{(T-N)N(1-2n)\sigma^2_v + (1-n)N\left(\sigma^2_{\alpha Ft}+\sigma^2_\varepsilon\right) - n(T-N)\left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)}{\left(2(T-N)N\sigma^2_v + N\left(\sigma^2_{\alpha Ft}+\sigma^2_\varepsilon\right) + (T-N)\left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)\right)}.$$

We have

$$\frac{\partial\Psi}{\partial\sigma^2_{\alpha Ft}} = \frac{1}{(1-n)n}\sum_{N=0}^{T} P(N|n)\, N\frac{N\sigma^2_v(T-N) + (T-N)\left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)}{\left(2(T-N)N\sigma^2_v + N\left(\sigma^2_{\alpha Ft}+\sigma^2_\varepsilon\right) + (T-N)\left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)\right)^2} > 0,$$

and hence using $\sigma^2_{\alpha Dt}$ in place of $\sigma^2_{\alpha Ft}$ in Ψ, we obtain:

$$\Psi \geq \frac{1}{(1-n)n}\sum_{N=0}^{T} P(N|n)\frac{(T-N)N(1-2n)\sigma^2_v + (N-Tn)\left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)}{\left(2(T-N)N\sigma^2_v + T\left(\sigma^2_{\alpha Dt}+\sigma^2_\varepsilon\right)\right)}.$$

By Lemma 5, the last expression is positive for $n < \frac{1}{2}$; hence, $\Psi > 0$ for $n \in \left(n', \frac{1}{2}\right)$. For $n = \frac{1}{2}$, the difference between the second expression in $\mu\Delta_t\left(\bar{y}^F; \mu, n, \bar{y}^D, \bar{y}^F\right)$ and the second expression in $\mu\Delta_t\left(\bar{y}^D;\mu,n,\bar{y}^D,\bar{y}^F\right)$ is

$$\begin{array}{rl} & \frac{1}{2\gamma}\left(\sum_{N=1}^{T-1}\frac{P\left(N|\frac{1}{2}\right)(2N-T)}{2(T-N)N\sigma_v^2+N\left(\sigma_{\alpha Ft}^2+\sigma_\varepsilon^2\right)+(T-N)\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)}+\sum_{N=0}^{T-1}P\left(N|\frac{1}{2}\right)\frac{T-N-1}{\sigma_{\alpha Ft}^2+\sigma_\varepsilon^2}-P\left(N|\frac{1}{2}\right)\frac{N-1}{\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2}\right) \\ < & \sum_{N=1}^{T-1}P\left(N|\frac{1}{2}\right)\frac{1}{2\gamma}\frac{2N-T}{2(T-N)N\sigma_v^2+N\left(\sigma_{\alpha Ft}^2+\sigma_\varepsilon^2\right)+(T-N)\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)}+\sum_{N=0}^{T}P\left(N|\frac{1}{2}\right)\frac{1}{2\gamma}\frac{T-2N}{\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2} \\ = & \sum_{N=1}^{T-1}P\left(N|\frac{1}{2}\right)\frac{1}{2\gamma}\frac{2N-T}{2(T-N)N\sigma_v^2+N\left(\sigma_{\alpha Ft}^2+\sigma_\varepsilon^2\right)+(T-N)\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)}<0. \end{array}$$

Hence, by the continuity of these expressions in n, there exists $n''<\frac{1}{2}$ such that the second expression in $\mu\Delta_t\left(\bar{y}^F;\mu,n,\bar{y}^D,\bar{y}^F\right)$ is smaller than the second expression in $\mu\Delta_t\left(\bar{y}^D;\mu,n,\bar{y}^D,\bar{y}^F\right)$ for all $n\in\left(n'',\frac{1}{2}\right)$. Moreover, the second expression in $\mu\Delta_t\left(\bar{y}^D;\mu,n,\bar{y}^D,\bar{y}^F\right)$ is negative, as the following calculation shows:

$$\begin{array}{rl} & \sum_{N=1}^{T}P(N|n)\left(\frac{(N+n-1)(T-N)}{2N(T-N)\sigma_v^2+N\left(\sigma_{\alpha Ft}^2+\sigma_\varepsilon^2\right)+(T-N)\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)}-\frac{(1-n)(N-1)}{\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)}\right) \\ < & \sum_{N=1}^{T}P(N|n)\left(\frac{(N+n-1)(T-N)}{T\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)}-\frac{(1-n)(N-1)}{\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)}\right) \\ = & \sum_{N=0}^{T}P(N|n)\frac{N(Tn-N+1-n)}{T\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)}=\frac{Tn(Tn+1-n)-\left(Tn-Tn^2+(Tn)^2\right)}{T\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)}=0\ . \end{array}$$

Hence, $\Delta_t\left(\bar{y}^D;\mu,n,\bar{y}^D,\bar{y}^F\right)=\Delta_t\left(\bar{y}^F;\mu,n,\bar{y}^D,\bar{y}^F\right)>0$ if $\bar{y}^D-\bar{y}^F=0$, and both are decreasing in $\bar{y}^D-\bar{y}^F$, but $\Delta_t\left(\bar{y}^D;\mu,n,\bar{y}^D,\bar{y}^F\right)$ decreases more slowly. Hence, we can find $\bar{y}^D-\bar{y}^F>0$, for which $\Delta\left(\bar{y}^D;\mu,n,\bar{y}^D,\bar{y}^F\right)>0>\Delta\left(\bar{y}^F;\mu,n,\bar{y}^D,\bar{y}^F\right)$ for all $n\in\left(n'',\frac{1}{2}\right)$. By the continuity of Δ, we can therefore find small enough c such that for all $n\in\left(n'',\frac{1}{2}\right)$, we have $\Delta\left(y_j;\mu,n,\bar{y}^D,\bar{y}^F\right)>0$ for each $y_j\in\left[\bar{y}^D-c,\bar{y}^D+c\right]$, and $0>\Delta\left(y_i;\mu,n,\bar{y}^D,\bar{y}^F\right)$ for each $y_i\in\left[\bar{y}^F-c,\bar{y}^F+c\right]$. In other words, we can find $\bar{y}^D-\bar{y}^F>0$ for which condition (26) is satisfied for all $n\in\left(n'',\frac{1}{2}\right)$.

From (21) and (22), we know that profits are inversely proportional to μ. Hence, if $\Pi_t^M\left(y_j;\mu,n,\bar{y}^D,\bar{y}^F\right)>0$, one can guarantee profit exceeding the entry cost by setting μ small enough. Since profit is increasing in y_j, if we show that $\Pi_t^F\left(\bar{y}^F-c;\mu,n,\bar{y}^D,\bar{y}^F\right)>0$ for $t=1,2$, we can guarantee that conditions (27) and (28) are satisfied for small enough μ. Using equations (20) and (22), we obtain:

$$\Pi_t^F\left(y^F-c;\mu,n,\bar{y}^D,\bar{y}^F\right)=\frac{1}{\mu(1-n)}\sum_{N=0}^{T}P(N|n)\frac{\gamma\left((T-N)N\sigma_v^2+(T-N)\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)\right)+(T-N)N\left(\bar{y}^F-\bar{y}^D\right)}{\gamma\left(2(T-N)N\sigma_v^2+N\left(\sigma_{\alpha Ft}^2+\sigma_\varepsilon^2\right)+(T-N)\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)\right)}$$
$$-\frac{1}{\mu(1-n)}\sum_{N=0}^{T}P(N|n)\left(\frac{N}{\gamma\left(2(T-N)N\sigma_v^2+N\left(\sigma_{\alpha Ft}^2+\sigma_\varepsilon^2\right)+(T-N)\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)\right)}+\frac{T-N-1}{\gamma\left(\sigma_{\alpha Ft}^2+\sigma_\varepsilon^2\right)}\right)c.$$

Since in our construction we can pick c arbitrarily small, it suffices to show that

$$\sum_{N=0}^{T}P(N|n)\frac{T-N}{1-n}\frac{\gamma\left(\sigma_v^2+\frac{1}{N}\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)\right)+\bar{y}^F-\bar{y}^D}{\gamma(T-N)\left(2\sigma_v^2+\frac{1}{T-N}\left(\sigma_{\alpha Ft}^2+\sigma_\varepsilon^2\right)+\frac{1}{N}\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)\right)}>0, \tag{43}$$

for $t=1,2$. Clearly, this is satisfied for small enough $\bar{y}^D-\bar{y}^F>0$. Hence, what remains for us to show is that we can find $\bar{y}^D-\bar{y}^F>0$ small enough that the above expression is positive for $t=1,2$, but large enough so that $\Delta\left(\bar{y}^D;\mu,n,\bar{y}^D,\bar{y}^F\right)>0>\Delta\left(\bar{y}^F;n,\bar{y}^D,\bar{y}^F\right)$. Using (42) and (43), it follows that we can find such $\bar{y}^D - \bar{y}^F > 0$, if the following holds:

$$\sum_{t=1,2} \frac{\sum_{N=0}^{T} P\left(N|n\right) \frac{(T-N)N(1-2n)\sigma_v^2+(1-n)N\left(\sigma_{\alpha Ft}^2+\sigma_\varepsilon^2\right)-n(T-N)\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)}{2(T-N)N\sigma_v^2+N\left(\sigma_{\alpha Ft}^2+\sigma_\varepsilon^2\right)+(T-N)\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)}}{\sum_{N=1}^{T} P\left(N|n\right)\left(\frac{(N-1)(1-n)}{\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)} - \frac{(N+n-1)(T-N)}{2(T-N)N\sigma_v^2+N\left(\sigma_{\alpha Ft}^2+\sigma_\varepsilon^2\right)+(T-N)\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)}\right)}$$

$$< \sum_{t=1,2} \frac{\sum_{N=0}^{T} P\left(N|n\right) \frac{N(T-N)\left(\sigma_v^2+\frac{1}{N}\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)\right)}{2(T-N)N\sigma_v^2+N\left(\sigma_{\alpha Ft}^2+\sigma_\varepsilon^2\right)+(T-N)\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)}}{\sum_{N=0}^{T} P\left(N|n\right) \frac{N(T-N)}{2(T-N)N\sigma_v^2+N\left(\sigma_{\alpha Ft}^2+\sigma_\varepsilon^2\right)+(T-N)\left(\sigma_{\alpha Dt}^2+\sigma_\varepsilon^2\right)}}.$$

By the continuity of the above expressions in n, it is enough to show that the above is satisfied for $n = \frac{1}{2}$. Moreover, for $n = \frac{1}{2}$, the expression for $t = 1$ on left-hand side is 0. Hence, it is enough to show that

$$\frac{\sum_{N=0}^{T} P\left(N|\frac{1}{2}\right) \frac{N\left(\sigma_{\alpha F2}^2+\sigma_\varepsilon^2\right)-(T-N)\left(\sigma_{\alpha D2}^2+\sigma_\varepsilon^2\right)}{2(T-N)N\sigma_v^2+N\left(\sigma_{\alpha F2}^2+\sigma_\varepsilon^2\right)+(T-N)\left(\sigma_{\alpha D2}^2+\sigma_\varepsilon^2\right)}}{\frac{\frac{T}{2}-1}{\sigma_{\alpha D2}^2+\sigma_\varepsilon^2} - \sum_{N=1}^{T} P\left(N|\frac{1}{2}\right) \frac{(2N-1)(T-N)}{2(T-N)N\sigma_v^2+N\left(\sigma_{\alpha F2}^2+\sigma_\varepsilon^2\right)+(T-N)\left(\sigma_{\alpha D2}^2+\sigma_\varepsilon^2\right)}}$$

$$< \frac{\sum_{N=0}^{T} P\left(N|\frac{1}{2}\right) \frac{N(T-N)\left(\sigma_v^2+\frac{1}{N}\left(\sigma_{\alpha D1}^2+\sigma_\varepsilon^2\right)\right)}{2(T-N)N\sigma_v^2+T\left(\sigma_{\alpha D1}^2+\sigma_\varepsilon^2\right)}}{\sum_{N=0}^{T} P\left(N|\frac{1}{2}\right) \frac{N(T-N)}{2(T-N)N\sigma_v^2+T\left(\sigma_{\alpha D1}^2+\sigma_\varepsilon^2\right)}} + \frac{\sum_{N=0}^{T} P\left(N|\frac{1}{2}\right) \frac{N(T-N)\left(\sigma_v^2+\frac{1}{N}\left(\sigma_{\alpha D2}^2+\sigma_\varepsilon^2\right)\right)}{2(T-N)N\sigma_v^2+N\left(\sigma_{\alpha F2}^2+\sigma_\varepsilon^2\right)+(T-N)\left(\sigma_{\alpha D2}^2+\sigma_\varepsilon^2\right)}}{\sum_{N=0}^{T} P\left(N|\frac{1}{2}\right) \frac{N(T-N)}{2(T-N)N\sigma_v^2+N\left(\sigma_{\alpha F2}^2+\sigma_\varepsilon^2\right)+(T-N)\left(\sigma_{\alpha D2}^2+\sigma_\varepsilon^2\right)}}.$$

By partially differentiating both sides with respect to σ_v^2, it is easy to show that the left-hand side is decreasing in σ_v^2, while the right-hand side is increasing in σ_v^2. Hence, it suffices to show that the above holds for $\sigma_v^2 = 0$. Rearranging terms, and using the fact that $\sigma_{\alpha D1}^2 > \sigma_{\alpha D2}^2$, we find that the above is satisfied if the expression below holds:

$$\frac{\sum_{N=0}^{T} P\left(N|\frac{1}{2}\right) \frac{N\left(\sigma_{\alpha F2}^2+\sigma_\varepsilon^2\right)-(T-N)\left(\sigma_{\alpha D2}^2+\sigma_\varepsilon^2\right)}{N\left(\sigma_{\alpha F2}^2+\sigma_\varepsilon^2\right)+(T-N)\left(\sigma_{\alpha D2}^2+\sigma_\varepsilon^2\right)}}{\frac{T}{2}-1-\sum_{N=1}^{T} P\left(N|\frac{1}{2}\right) \frac{(2N-1)(T-N)\left(\sigma_{\alpha D2}^2+\sigma_\varepsilon^2\right)}{N\left(\sigma_{\alpha F2}^2+\sigma_\varepsilon^2\right)+(T-N)\left(\sigma_{\alpha D2}^2+\sigma_\varepsilon^2\right)}} < \frac{2}{(T-1)} + \frac{\sum_{N=0}^{T} P\left(N|\frac{1}{2}\right) \frac{(T-N)\left(\sigma_{\alpha D2}^2+\sigma_\varepsilon^2\right)}{N\left(\sigma_{\alpha F2}^2+\sigma_\varepsilon^2\right)+(T-N)\left(\sigma_{\alpha D2}^2+\sigma_\varepsilon^2\right)}}{\sum_{N=0}^{T} P\left(N|\frac{1}{2}\right) \frac{N(T-N)\left(\sigma_{\alpha D2}^2+\sigma_\varepsilon^2\right)}{N\left(\sigma_{\alpha F2}^2+\sigma_\varepsilon^2\right)+(T-N)\left(\sigma_{\alpha D2}^2+\sigma_\varepsilon^2\right)}}.$$

Evaluating both sides at $\sigma_{\alpha F2}^2 = \sigma_{\alpha D2}^2$ and using L'Hospital's rule, we obtain:

$$\frac{2}{T} < 2\frac{2}{(T-1)}.$$

Hence, for an open set of parameters for which $\sigma_{\alpha F2}^2$ is close to $\sigma_{\alpha D2}^2$, the desired inequality holds.[23]

And finally, (19)–(22) say that the expected profit from entering a particular asset market is linearly increasing in y_j. Hence, for any $\left\{\mu, n, \bar{y}^D, \bar{y}^F\right\}$ and any (F_D, F_F), we can always find y^L so low, that any $y_j \leq y^L + c$ expects a negative profit from any specialization; hence, condition (29) is satisfied.

Proof of Remark 1. From the proof of Proposition 3, we know that $\bar{y}^F < \bar{y}^D$, which could not be an equilibrium if $\frac{\partial\Delta\left(y_j;\mu,n,\bar{y}^D,\bar{y}^F\right)}{\partial y_j} < 0$. Hence, either $\frac{\partial\Delta\left(y_j;\mu,n,\bar{y}^D,\bar{y}^F\right)}{\partial y_j} = 0$, and all operating managers are indifferent between the assets, or there there exists $\hat{y}_d$ for which $\Delta\left(\hat{y}_d;\mu,n,\bar{y}^D,\bar{y}^F\right) = 0$. The existence of $\hat{y}_o$ follows from the fact that for $M = D, F$, the

[23] Simulations show that the inequality holds for any $\sigma_{\alpha F2}^2 > \sigma_{\alpha D2}^2$.

expression $\Pi_t^M\left(y_j;\mu,n,\bar{y}^D,\bar{y}^F\right)$ is strictly increasing in y_j. ■

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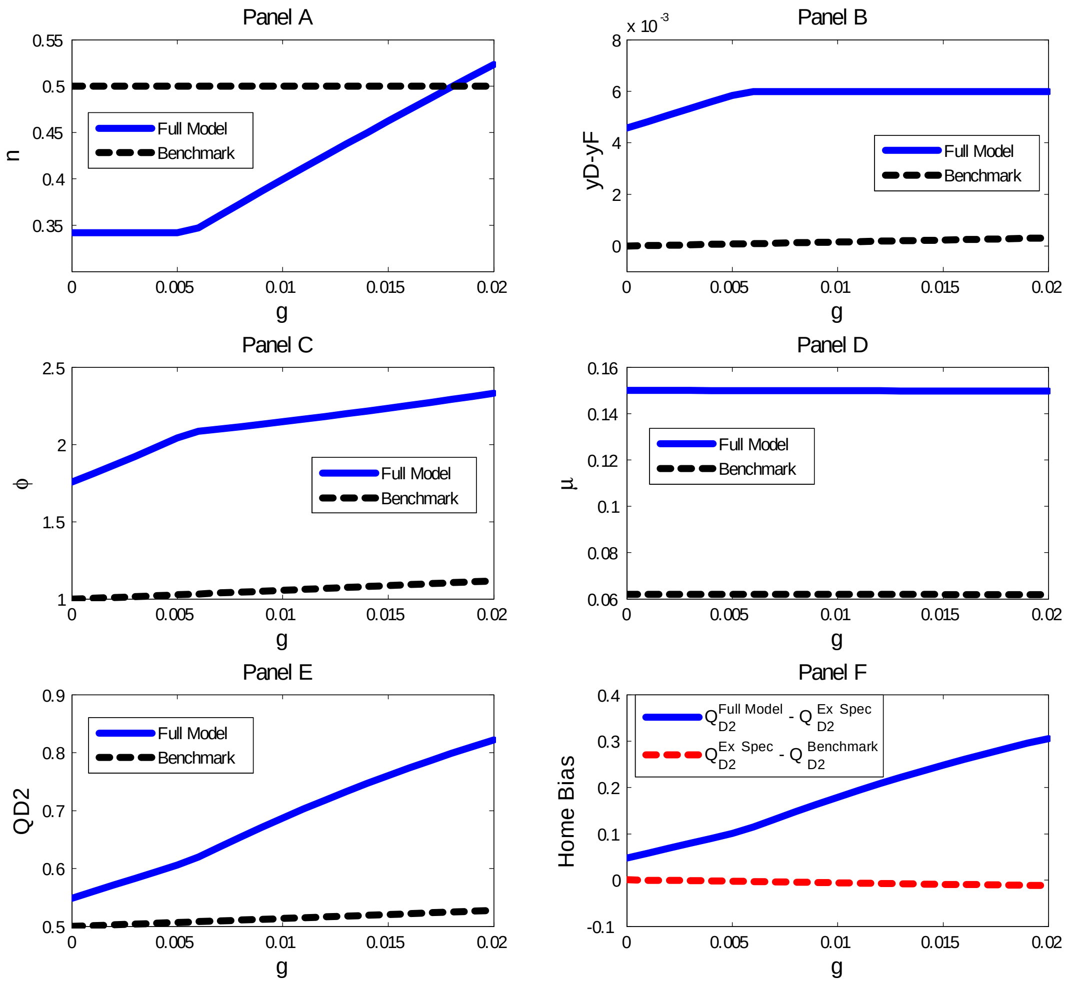

Figure 1: This figure shows comparative statics with respect to g for the full model (solid line) and the benchmark model (dashed line). Panel A plots the equilibrium fraction of managers investing domestically, n. Panel B shows the difference between the average ability of domestic-asset and foreign-asset managers, $y^D - y^F$. Panel C presents the market value of domestic-asset funds over foreign-asset funds in the second period, $\phi \equiv \frac{(Q_{D1}+Q_{D2})/n}{(Q_{F1}+Q_{F2})/(1-n)}$. Panel D reports the fraction of operating managers, μ. Panel E exhibits the total expected amount of capital channeled to the domestic assets in the second period, Q_{D2}. Panel F decomposes the home bias into $Q_{D2}^{Full\ Model} - Q_{D2}^{Ex\ Spec}$ and $Q_{D2}^{Ex\ Spec} - Q_{D2}^{Benchmark}$.

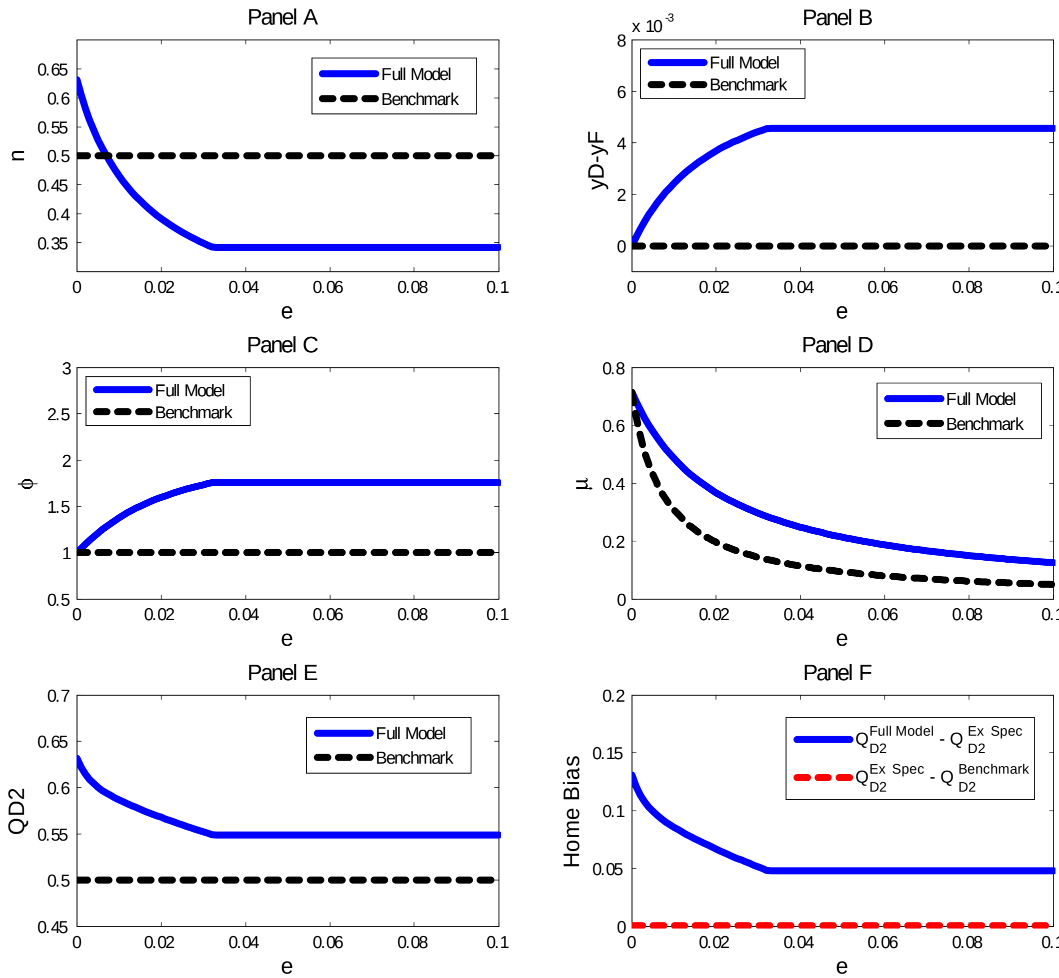

Figure 2: This figure shows comparative statics with respect to e for the full model (solid line) and the benchmark model (dashed line) when $g = 0$. Panel A plots the equilibrium fraction of managers investing domestically, n. Panel B shows the difference between the average ability of domestic-asset and foreign-asset managers, $y^D - y^F$. Panel C presents the market value of domestic-asset funds over foreign-asset funds in the second period, $\phi \equiv \frac{(Q_{D1}+Q_{D2})/n}{(Q_{F1}+Q_{F2})/(1-n)}$. Panel D reports the fraction of operating managers, μ. Panel E exhibits the total expected amount of capital channeled to the domestic assets in the second period, Q_{D2}. Panel F decomposes the home bias into $Q_{D2}^{Full\ Model} - Q_{D2}^{Ex\ Spec}$ and $Q_{D2}^{Ex\ Spec} - Q_{D2}^{Benchmark}$.

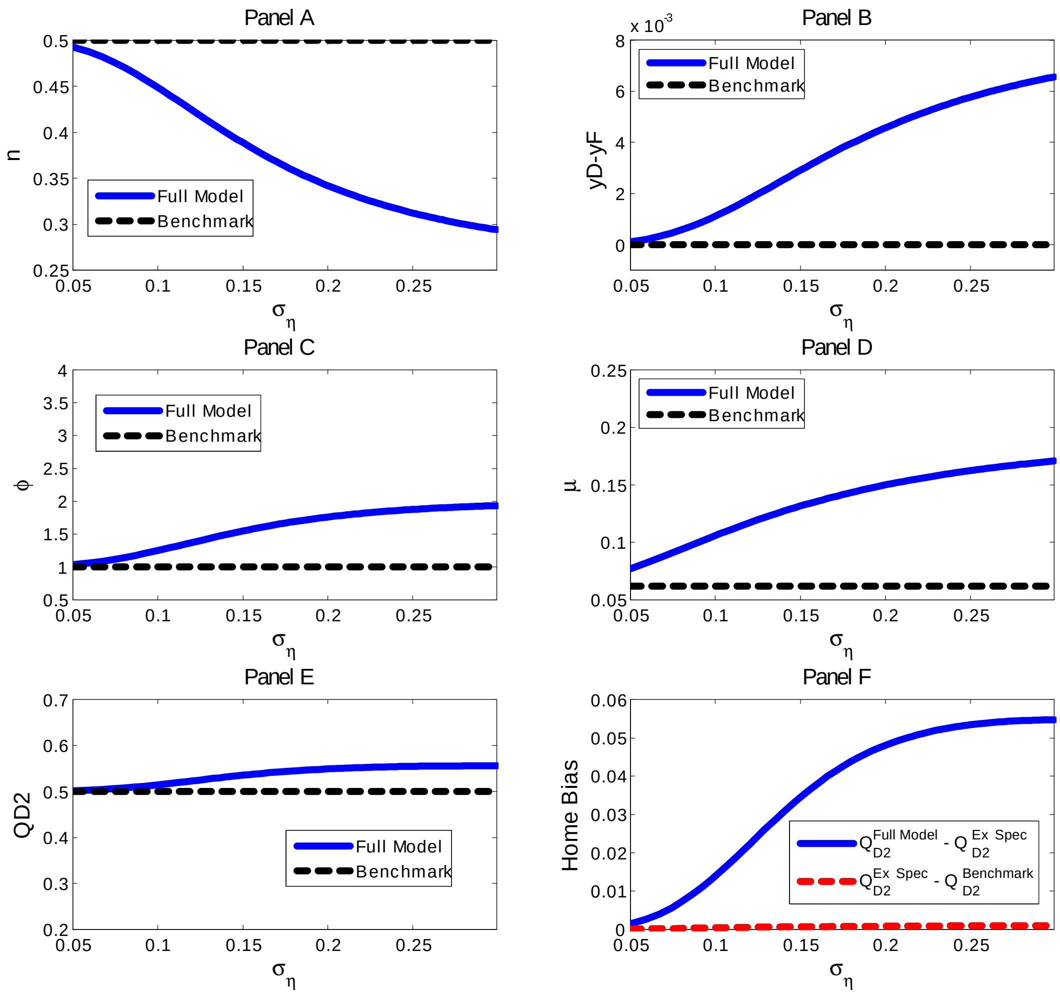

Figure 3: This figure shows comparative statics with respect to σ_η for the full model (solid line) and the benchmark model (dashed line) when $g = 0$. Panel A plots the equilibrium fraction of managers investing domestically, n. Panel B shows the difference between the average ability of domestic-asset and foreign-asset managers, $y^D - y^F$. Panel C presents the market value of domestic-asset funds over foreign-asset funds in the second period, $\phi \equiv \frac{(Q_{D1}+Q_{D2})/n}{(Q_{F1}+Q_{F2})/(1-n)}$. Panel D reports the fraction of operating managers, μ. Panel E exhibits the total expected amount of capital channeled to the domestic assets in the second period, Q_{D2}. Panel F decomposes the home bias into $Q_{D2}^{Full\ Model} - Q_{D2}^{Ex\ Spec}$ and $Q_{D2}^{Ex\ Spec} - Q_{D2}^{Benchmark}$.

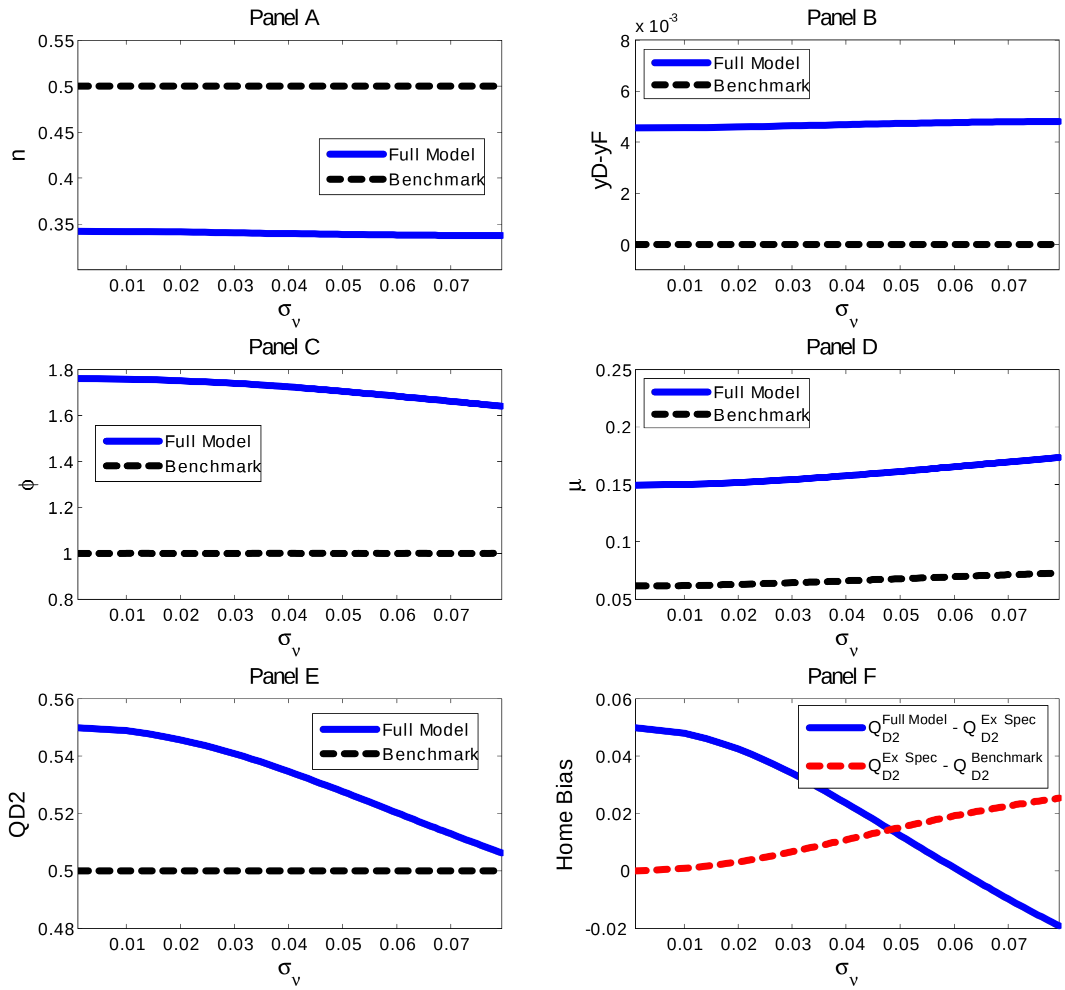

Figure 4: This figure shows comparative statics with respect to σ_ν for the full model (solid line) and the benchmark model (dashed line) when $g = 0$. Panel A plots the equilibrium fraction of managers investing domestically, n. Panel B shows the difference between the average ability of domestic-asset and foreign-asset managers, $y^D - y^F$. Panel C presents the market value of domestic-asset funds over foreign-asset funds in the second period, $\phi \equiv \frac{(Q_{D1}+Q_{D2})/n}{(Q_{F1}+Q_{F2})/(1-n)}$. Panel D reports the fraction of operating managers, μ. Panel E exhibits the total expected amount of capital channeled to the domestic assets in the second period, Q_{D2}. Panel F decomposes the home bias into $Q_{D2}^{Full\ Model} - Q_{D2}^{Ex\ Spec}$ and $Q_{D2}^{Ex\ Spec} - Q_{D2}^{Benchmark}$.

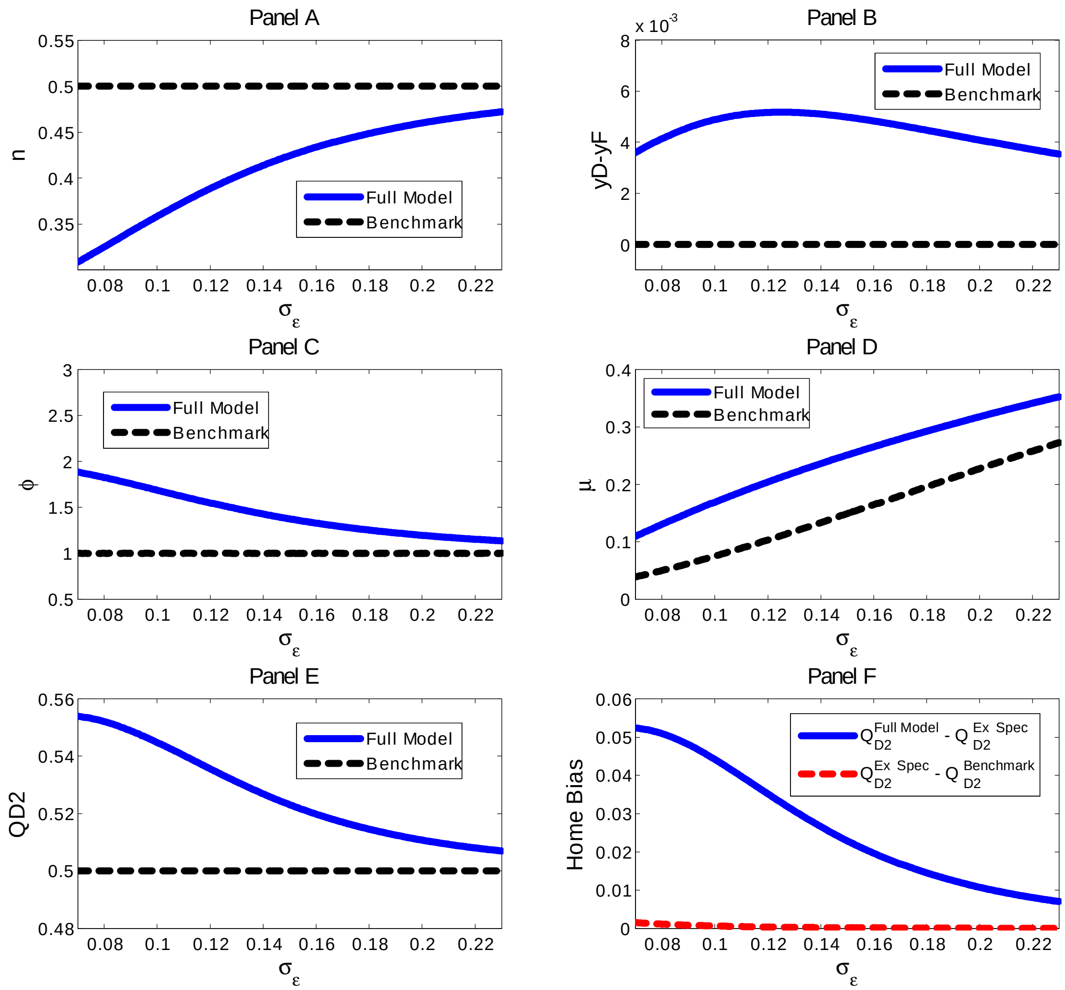

Figure 5: This figure shows comparative statics with respect to σ_ε for the full model (solid line) and the benchmark model (dashed line) when $g = 0$. Panel A plots the equilibrium fraction of managers investing domestically, n. Panel B shows the difference between the average ability of domestic-asset and foreign-asset managers, $y^D - y^F$. Panel C presents the market value of domestic-asset funds over foreign-asset funds in the second period, $\phi \equiv \frac{(Q_{D1}+Q_{D2})/n}{(Q_{F1}+Q_{F2})/(1-n)}$. Panel D reports the fraction of operating managers, μ. Panel E exhibits the total expected amount of capital channeled to the domestic assets in the second period, Q_{D2}. Panel F decomposes the home bias into $Q_{D2}^{Full\ Model} - Q_{D2}^{Ex\ Spec}$ and $Q_{D2}^{Ex\ Spec} - Q_{D2}^{Benchmark}$.