# The Asset Pricing and Real Implications of Relationship Intensity Disclosure

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**Abstract**

In many scenarios, investors in financial markets are uncertain about the relationship intensity between two firms and have to rely on firms' disclosure of such relationship intensity. We develop a theory to study the asset pricing implications of this relationship intensity uncertainty and how such relationship intensity uncertainty affects firms' incentives to form and disclose their relationship intensities to the public in the first place (i.e., the real implications). We find that while such disclosure has a positive price impact by increasing the expected cash flow, it also has a negative price impact by reducing the diversification benefit (or, equivalently, increasing the diversification cost) of investing in multiple firms that have more correlated cash flows. The price impact upon relationship intensity disclosure is

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therefore not monotone: it increases with the expected benefit of relationship and decreases with the risk of the underlying relationship. One main policy implication of our analysis is that mandatory disclosure of firm relationship intensities may both destroy relationship development and reduce investor welfare. In other words, disclosing relationship intensity information can have real consequences on cash flows through affecting firm relationships at both the intensive and the extensive margins. The results are robust to a battery of extensions.

**Keywords**: firm relationships, asset prices, disclosure, matching intensity, matching profitability

**JEL**: D82, G14, G18, M41, M45

# 1 Introduction

In this paper we propose a model that studies the asset pricing and real implications of disclosure about relationship intensity between firms.[1] Firms do not operate in isolation and financial markets consist of a network of companies that are linked to and interact with each other. These companies are linked through contractual relationships such as customer-supplier agreements or implicit relationships such as strategic alliances, common production or labor exposures, and similar regulation requirements or litigation risks. Not surprisingly, regulators require many disclosures of such linkages. For example, SFAS No. 131 requires firms to disclose the existence of and the total amount of revenues from each major customer that represents more than 10 percent of a firm's sales revenue. As another example, Regulation S-K requires firms to disclose all material contracts or agreements into which a firm enters. Such relationship disclosure will clearly affect investors' perception of the firms' underlying cash flows and thus their capital market prices. Concerns about capital market prices will in turn have real effects on firms' investment in such relationship in the first place (Kanodia and Sapra (2016)). In other words, disclosure of relationship affects the correlation of firms' cash flows endogenously.

Traditional theories on asset prices and disclosure assume that the correlation structure of asset payoffs is exogenously given and known to investors through links among firms. However, knowledge of such payoffs is often obtained through firms' endogenous disclosure choices. In addition, disclosure of such relationship is often incomplete and differs among firms. For instance, while regulation SFAS No. 131 requires firms to disclose the existence of and the percentage of revenues from each major customer that represents more than 10 percent of a firm's sales revenue, firms can choose whether to disclose the identity of the major customer. For example, Pepsi disclosed in its 2020 annual report that "Walmart and its affiliates (including Sam's) represented approximately 14% of our consolidated net revenue," whereas Nike only disclosed in its 2020 annual report that "our three largest customers accounted for approximately 24% of sales in the

[1] As will be discussed in more detail in Section 3, disclosing whether a relationship exists is a special case of disclosing relationship intensity since no relationship corresponds to the case of relationship intensity being zero. For simplicity we will use the term "relationship disclosure" when we actually mean "relationship intensity disclosure".

United States" without disclosing the names of those customers. In addition, as discussed in Ellis et al. (2012), firms can choose to voluntarily disclose non-major customers. The possibility of disclosing some but not all relationships suggests that investors are uncertain about relationships and exposure between firms.

Uncertainty about such relationships and exposures implies that investors have imperfect knowledge about the correlation structure of asset payoffs. In this paper, we study how investors value assets when there is such uncertainty about relationships, and the incentives of firms to disclose and form such relationships. While previous studies on firms' disclosure of relationships (e.g., Ellis et al. (2012) and Verrecchia and Weber (2006)) focus mostly on how proprietary cost affects firms' disclosure choices, we abstract away from such proprietary cost and instead focus on how the residual uncertainty about relationships affects firms' disclosure choices and, perhaps more importantly, firms' choices of whether to establish a relationship in the first place, through affecting firms' stock prices.

In our setting, there are two firms with an uncertain exposure to a common relationship risk factor, which we label as "matching intensity."[2] We assume that if two firms form a high (low) intensity match, their collaboration will generally lead to more (less) cash flows (in expectation). The valuation of both firms is determined by a representative investor who forms her portfolio in a competitive asset market. Having a relationship has two effects on the firms' valuation. First, the cash flows have an additional payoff component with a positive mean (i.e., firms on average collaborate on positive NPV projects). This increase in the mean of asset payoffs increases asset prices via boosting the investor's perceived returns on investing in those risky assets. Second, the cash flows of the two firms are more risky and become more correlated, which harms the investor's ability to diversify her portfolio, thereby lowering her demand for those risky assets and their prices.

The effect of the relationship on firms' prices in turn affects the optimal disclosure policy about

[2]Therefore our setting can be literally interpreted as two big firms in a large economy, which may not be unreasonable in the example of Walmart being a major customer of Pepsi, in addition to making the model tractable. We discuss how our results can be extended in settings with N-firms in Section 6.

matching intensity. Unlike most of the existing studies on disclosure, disclosure in our setting is about disclosing the exposure to a common component in the asset payoffs of the two firms, which generates the correlation structure between the two firms. It is not about disclosing the realization of the fundamentals as in most research on disclosure (see, e.g., Dye (2001),Verrecchia (2001), Beyer et al. (2010), Stocken (2013), Kanodia and Sapra (2016), and Goldstein and Yang (2017) for excellent reviews) and in particular, the literature on proprietary cost (Verrecchia (1983)). Even when abstracting away from such proprietary cost, we find that firms may not always want to make relationship disclosures. Specifically, firms in relationships with very large or very small matching profitability relative to the collaboration risk of forming such relationships will choose to fully disclose their relationships, while firms with matching profitability at an intermediate level relative to the collaboration risk will choose not to disclose any information about their relationship. The results are driven by the effect of more precise disclosure on higher moments of cash flow distribution, specifically, skewness and kurtosis, which relates our finding to that in Heinle et al. (2018). As discussed in more detail in Section 2, our binary distribution of relationship intensity implies that more precise disclosure does not unambiguously increase skewness and kurtosis.

Loosely speaking, the matching profitability determines the magnitude of the impact of the skewness of the cash flow distribution on price whereas the collaboration risk determines the impact of the degree of kurtosis of the cash flow distribution on price. When matching profitability is relatively high compared with collaboration risk, more precise disclosure about collaboration quality both increases the asymmetry of the distribution thus skewness (which increases prices) and increases kurtosis (which decreases prices), as the high matching profitability magnifies both the asymmetry of the distribution and the downside risk, but the impact of the skewness term dominates, resulting in full disclosure being optimal. When matching profitability is in the middle, for the same reason more precise disclosure increases skewness and kurtosis, but the kurtosis term dominates, resulting in no disclosure being optimal. When the matching profitability is relatively low compared with collaboration risk, the cash flow distribution exhibits relatively high tail risk. More precise disclosure of matching profitability reduces kurtosis by reducing tail risk (which increases prices), and also reduces the asymmetry of the distribution and thus skewness (which reduces prices) through the tail risk reduction, as the tail risk contributes to the asymmetry in distribution. With the kurtosis term dominating, full disclosure is again optimal.

We then use our framework to investigate the choice of firms to form relationships and how the choice of forming relationships interacts with the choice of disclosing such relationship, that is, the real effects of such disclosure. Unlike previous studies, we do not take the relationship as given but we treat the decision to form a relationship as endogenous. Firms will choose to form a relationship when the expected price of forming a relationship is higher than the expected price when there is no relationship. We assume the decision is made before the firm is aware whether the relationship will be a high intensity or low intensity match. We find that when the expected benefits of forming a relationship are very large relative to the costs, firms choose to form a relationship with disclosure. Instead, when the expected benefits are at intermediate levels relative to the costs, firms form a relationship without disclosure. Intuitively, when the expected benefit is sufficiently high, forming a relationship increases the perceived firm value and thus expected price. Conditional on forming a relationship, whether or not firms would want to disclose depends on the relative magnitude of the expected benefit, as discussed above. Finally, when the relative expected benefits are low, firms choose not to form a relationship.

This new setting allows us to analyze the implications of policies that mandate disclosure of relationships. We find that even absent proprietary cost concerns, mandatory disclosure may prevent relationship formation. Specifically, we find that when firms are forced to disclose their relationships, some relationships that would have been formed, specifically, when the expected benefits of forming a relationship relative to the costs are at an intermediate range, will not be formed at all. The reason is that, as discussed above, firms would voluntarily choose not to disclose in this case. Therefore, mandatory disclosure, through reducing expected prices, results in firms not forming relationships in the first place. Thus, mandatory disclosure will have an effect on the extensive margin of relationships. In addition, we show that such real effects have an unambiguously negative effect on the welfare of the representative investor, as the reduced expected cash flow more than compensates the reduced risk from not forming a (risky) relationship in the first place. In contrast, mandatory disclosure have an unambiguously positive effect on the welfare of the representative investor when relationship formation is not destroyed, by lowering the price that investors pay for firms. Our paper therefore provides a novel trade-off regarding regulation aimed at relationship disclosure such as SFAS 131.

Finally, we analyze a couple of extensions of the model. We first extend our basic framework to settings of N firms, where each firm can choose whether to form a relationship with the rest of N-1 firms. We first find that, counter-intuitively, that even though the relationships are idiosyncratic, the relationship risk does not vanish even in a large economy. The reason is that as $N$ becomes large, each firm forms $N-1$ independent relations with other firms, and so each firm's risk increases exactly at the same speed as firm numbers. As a result, the risk of a representative investor's equilibrium portfolio remains a constant and does not vanish. We also numerically verify that our results in the basic framework remain qualitatively unchanged: firms would voluntarily disclose their relationship if the matching profitability relative to collaboration risk is either sufficiently large or small, and mandatory relationship destroys such relationship if firms' matching profitability relative to the collaboration risk is in the intermediate range.

We also consider a setting where there is correlation between firm-specific cash flow and cash flow from relationship formation. We find that firms are more likely to disclose relationship when the correlation is negative and less likely when the correlation is positive. Intuitively, negative (positive) correlation reduces (increases) the diversification cost, making firms more likely to disclose. Nevertheless, our results in the basic framework remain qualitatively unchanged so long as the correlation is not too negative, as firms will always disclose their relationships when the correlation becomes too negative.

Finally, we conduct a complete welfare analysis of mandatory relationship intensity disclosure following Diamond (1985). We find that such mandatory disclosure does not have any welfare benefit. If the disclosure does not destroy firms' relationship formation, it does not affect firms' real decisions and thus welfare remains unchanged. However, if such disclosure destroys firms' relationship formation, then the negative real effect strictly reduces investors' welfare.

The rest of this paper is organized as follows. Section 2 discusses related literature. Section 3 introduces our framework to study the asset pricing implications regarding uncertainty of a relationship between two firms. Section 4 addresses firms' voluntary disclosure strategies of relationships. Section 5 discusses how relationship disclosure affects formation of relationship in the first place, that is, the real implications of relationship disclosure. Section 6 discusses extensions to N firms. Section 7 introduces correlation between firm-specific cash flow and cash flow from relationship formation. Section 8 examines the effect of disclosure mandate on the representative investor's welfare. Section 9 discusses implications of our analysis. Finally, Section 10 concludes. All proofs are relegated to the Appendix. More technical details and variations of the model are included in the Online Appendix.

## 2 Related Literature

Our paper is related to the extensive literature that has examined the implications of disclosure for market quality. Recent studies analyze the implications of financial disclosure and argues that both the amount (e.g.,Tang (2014)) and the type of the information disclosed (e.g., Goldstein and Yang (2019)) are crucial determinants of market quality (see Verrecchia (2001), Dye (2001), and, more recently, Goldstein and Yang (2017) for surveys on this topic). The main reason to disclose information voluntarily is to reduce information asymmetries between investors and firms (e.g., Diamond and Verrecchia (1991), Easley and O'hara (2004), Lambert et al. (2007)) or reducing uncertainty about future payoffs (e.g., Barry and Brown (1985), Coles and Loewenstein (1988), Cheynel (2013)). The implications of mandatory disclosure are studied by Fishman and Hagerty (2003) who show that customers' sophistication level is of great importance to determine the benefit of mandatory disclosure, and by the real effects literature (Kanodia and Sapra (2016)) showing that mandatory disclosure may have unintended consequences of distorting firms' real decisions. Unlike these prior studies, where disclosure is related to the realization of private information, our paper focuses on disclosure of firm relationships and its implications for asset prices.

Our paper is also related to the vast literature on diversification discount (e.g., Lang and Stulz (1994), Berger and Ofek (1995)), which focuses on the cost of having multiple segments inside a firm instead of forming a relationship between firms. While this literature in general documents a discount for diversified firms relative to the sum of the estimated value of their segments, Campa and Kedia (2002) show that after controlling for the endogenous decision of firms to diversify, diversification actually improves firm value, which is similar in spirit to our assumption that forming relationship improves firm value. However, the diversification literature finds that diversification within a firm reduces risk, whereas we find that relationship formation increases the systematic risk. This result is consistent with the empirical findings in Herskovic (2018), who documents that networks in production (a source of inter-firm relationship) are sources of systematic risk reflected in equilibrium asset prices.

Another line of research studies capital market response to firms' disclosure choice about certain relationships, and how the existence of such relationships affect firms' public disclosures. Examples include disclosure about consumers (Ellis et al. (2012)), material contracts (Verrecchia and Weber (2006), Samuels (2021)), and strategic alliances (Ma (2019), Kepler (2021)). Analytically, Darrough and Stoughton (1990) and Wagenhofer (1990) model the decision to disclose information on customer-supplier relationships when there is a benefit of reducing information asymmetries but also a cost of revealing proprietary information to competitors. Correspondingly, most if not all of the papers are couched in a proprietary cost framework, suggesting that firms should always disclose such relationships in the absence of proprietary cost. In contrast, our paper shows that even without proprietary cost, firms may not want to make relationship disclosure because such disclosure results in a diversification cost. In addition, our paper also studies the effect of disclosure on the firms' decision to form a relationship, whereas relationships are assumed to be exogenous in those papers.

In a closely related paper, Heinle et al. (2018) analyze the effects of factor-exposure uncertainty on asset prices and the implications of disclosure about exposure. Similarly, our paper models uncertainty about a relationship as uncertainty about exposure to a common factor between two firms. Since there are two firms in our model, we are able to capture a diversification cost of having a relationship that is not explored by Heinle et al. (2018). Also, the way we model exposure to the common factor is different because we are pursuing different research questions. Our model is able to capture a situation in which an investor does not know whether two firms have a relationship, while this is not possible in the previous literature on disclosure. In addition, as stated previously, our paper endogenizes the firms' decision to form a relationship instead of taking it as exogenously given. Of course, our paper also shares a lot of similarities, e.g., prices are affected by higher order moments of cash flow distribution and more disclosure does not necessarily increase the cost of capital. However, our paper also differs from Heinle et al. (2018) in this aspect, by showing that more precise disclosure does not unambiguously increase skewness and decrease kurtosis. We believe this is due to our binary distribution of factor exposure being not a symmetric distribution, as is the normal distribution in Heinle et al. (2018). In addition, our binary distribution provides us with a tractable framework to characterize the necessary and sufficient conditions for more precise disclosure to decrease the cost of capital, thus allowing us to answer the question of the real effects of more precise disclosure on relationship formation.[3]

A growing literature studies endogenous network formation, in which the aim of forming such network could be acquiring information (Herskovic and Ramos (2018), Galeotti and Goyal (2010)), or forming input-ouput relationships (Acemoglu and Azar (2017), Taschereau-Dumouchel (2017), Lim (2018), Oberfield (2018), Tintelnot et al. (2018)).[4] In contrast to these papers, disclosure policy is at the core of our analysis. Furthermore, our paper contributes to this literature by showing that disclosure policies can affect customer-supplier relationships. More specifically, mandatory disclosure may prevent such relationship formation in the first place.

[3]See Section 4 for more detailed discussions.

[4]See also Bernard and Moxnes (2018) and Carvalho and Tahbaz-Salehi (2018) for recent literature reviews on production networks in international trade and macroeconomics, respectively.

Finally, our paper features relationship uncertainty, which connects our framework to a few theoretical studies in the literature that have considered uncertain factor loadings in various forms (e.g., Armstrong et al., 2013; Beyer and Smith, 2021; Huang et al., 2021) and a more recent literature on disclosing algorithms (e.g., Brunnermeier et al. (2020), Sun (2021)). In these papers, the random factor loadings or the statistical properties of algorithms are exogenous, while our framework makes an effort to endogenize these random loadings via relationship formation.

# 3 Conceptual Framework

In this section, we build a framework to study the effects of uncertainty of a relationship between two firms on asset prices.

## 3.1 Setup

Consider two symmetric firms: $A$ and $B$. We normalize the number of shares of each firm to $1/2$. The cash flows of each firm have two components and are given by $\tilde{F}_A = \tilde{V}_A + \tilde{\Delta}$ and $\tilde{F}_B = \tilde{V}_B + \tilde{\Delta}$. The components $\tilde{V}_A$ and $\tilde{V}_B$ are firm-specific (and so mutually independent) and normally distributed with mean $\bar{V}$ and variance $\sigma_V^2$: $\tilde{V}_A \sim N(\bar{V}, \sigma_V^2)$ and $\tilde{V}_B \sim N(\bar{V}, \sigma_V^2)$. The component $\tilde{\Delta}$ is common between the two firms and reflects the cash flow correlation driven by the relationship between both firms. This second component is given by

$$\tilde{\Delta} = \tilde{\rho}\tilde{\delta}, \text{ with } \tilde{\delta} \sim N\left(\bar{\delta}, \sigma_\delta^2\right) \text{ and } \tilde{\rho} = \begin{cases} \rho_h & \text{with probability } \pi, \\ \rho_l & \text{with probability } 1 - \pi, \end{cases} \tag{1}$$

where $\rho_h > \rho_l \geq 0$ and $\bar{\delta} \geq 0$. The random variables $\tilde{\rho}, \tilde{\delta}, \tilde{V}_A$, and $\tilde{V}_B$ are mutually independent.

In this setup, a relationship between the two firms determines their payoff correlation through two elements, $\tilde{\rho}$ and $\tilde{\delta}$. First, a relationship may turn out to be a high intensity match with $\rho_h$ or a low intensity match with $\rho_l$. We refer to $\tilde{\rho}$ as "matching intensity." Second, given matching intensity, if the two firms engage in intensive collaboration, say, collaborating across multiple product lines, their payoff correlation tends to be strong. We interpret $\tilde{\delta}$, or more specifically, its mean $\bar{\delta}$, as "matching profitability." The relative magnitude of matching intensity can be empirically proxied by, e.g., revenue from the relationship as a percentage of the total revenue for a focal firm pair whereas the relative magnitude of matching profitability can be proxied by, e.g., the average revenue from relationship by firm pairs that lie in the same industry as the focal firm pairs, assuming that such averaging removes the variation in matching intensity.

A special case of specification (1) is $\pi = \rho_l = 0$, which implies that $\tilde{\Delta} = 0$. This degenerate setting corresponds to a benchmark setting with independent cash flows (no relationship between the two firms). As discussed in the introduction, another interesting but less specialized case is $\rho_l = 0$ but $\pi > 0$. In this case, the uncertainty about matching intensity $\tilde{\rho}$ can be reinterpreted as uncertainty about the existence of a relationship between firms: the market is ex ante uncertain about whether two firms are related, but once the market becomes certain that there is a relationship between the two firms, then it is known that the matching intensity of the two firms is $\rho_h$. Therefore, even our focus is on disclosing matching intensity, it can be applied to the discussion of disclosing whether a relationship exists or not.

There exists a representative investor, whose preference is $-e^{-\gamma \tilde{W}}$, where $\gamma > 0$ is the absolute risk aversion coefficient and $\tilde{W}$ is the final wealth. The investor is initially endowed with $W_0$ wealth and chooses the asset holdings that maximize her preference subject to the following budget constraint

$$\tilde{W} = W_0 R + q_A(\tilde{F}_A - RP_A) + q_B(\tilde{F}_B - RP_B), \tag{2}$$

where $R$ is the exogenous risk-free interest rate, $q_A$ and $q_B$ are the asset holdings of the risky assets, and $P_A$ and $P_B$ are asset prices. In the following two subsections, we compute asset prices when the matching intensity is known or random respectively.

As a final remark, we consider a two-firm setting as the main model, which can be literally interpreted as two big firms in their respective industries. A two-firm setting is the simplest way to model relationship among firms and is consistent with the Pepsi-Walmart example discussed in the introduction. While it certainly raises the issue of whether the risk exposure due to the relationship among two firms can be diversified away in a multiple-firm setting, we later in Section 6 extend our analysis to N-firm settings and show that our results remain largely the same, when relationship formation among N firms are modelled as a natural extension to that of the two-firm setting. In addition, the risk exposure due to the relationship among firms is not diversified away. For now we focus on the two-firm setting to better illustrate the driving forces and the underlying intuition of our results.

## 3.2 Asset Prices under Perfect Matching Information

We use superscript "PI" to denote the case in which the investor has perfect information about the firms' matching intensity $\tilde{\rho}$. That is, the representative investor knows the realization of $\tilde{\rho}$. In this case, the common component of cash flows $\tilde{\Delta}$ follows a Normal distribution. The expectation and variance of final wealth in equation (2) for any realization of $\tilde{\rho}$ are given by

$$E[\tilde{W} \mid \tilde{\rho}] = W_0 R + q_A(\bar{V} + \tilde{\rho}\bar{\delta} - RP_A^{PI}) + q_B(\bar{V} + \tilde{\rho}\bar{\delta} - RP_B^{PI}),$$

and

$$V[\tilde{W} \mid \tilde{\rho}] = q_A^2(\sigma_V^2 + \tilde{\rho}^2\sigma_\delta^2) + q_B^2(\sigma_V^2 + \tilde{\rho}^2\sigma_\delta^2) + 2q_A q_B \tilde{\rho}^2\sigma_\delta^2.$$

When the investor maximizes her expected utility subject to the budget constraint, her demand for asset $A$ is given by

$$q_A^{PI} = \frac{(\sigma_V^2 + \tilde{\rho}^2\sigma_\delta^2)(\bar{V} + \tilde{\rho}\bar{\delta} - RP_A^{PI}) - \tilde{\rho}^2\sigma_\delta^2(\bar{V} + \tilde{\rho}\bar{\delta} - RP_B^{PI})}{\gamma[(\sigma_V^2 + \tilde{\rho}^2\sigma_\delta^2)^2 - (\tilde{\rho}^2\sigma_\delta^2)^2]}. \tag{3}$$

Similarly, one can find the demand for asset $B$. The demand of one asset is affected by the demand of the other asset. We can find the asset prices under perfect information by imposing the market-clearing conditions $q_A = 1/2$ and $q_B = 1/2$. The price for asset $j \in \{A, B\}$ under perfect information is given by

$$P_j^{PI}(\tilde{\rho}) = \frac{\bar{V} - \frac{\gamma}{2}\sigma_V^2}{R} + \frac{\bar{\delta}}{R}\tilde{\rho} - \frac{\gamma\sigma_\delta^2}{R}\tilde{\rho}^2, \text{ for } j \in \{A, B\}. \tag{4}$$

The first term of the price would be the price when $\tilde{\rho} = \tilde{\Delta} = 0$, that is, when firms have no relationship. The second and third terms convey the additional effects of relationships in prices. Specifically, the second term captures the benefit of having a relationship, which is an additional payoff factor. The third term, instead, captures the cost of forming a relationship: payoffs of both assets are more risky and become correlated, making it more difficult for the investor to diversify risk. The risk aversion parameter $\gamma$ affects prices in two ways. First, the weight of the variance in the price increases with risk aversion. Second, the risk aversion parameter also affects the weight associated with the decrease in the investor's ability to diversify when firms form a relationship. Under both effects, prices decrease with risk aversion.

The next corollary characterizes how the features of a relationship influence asset prices under perfect matching information.

**Corollary 1.** *Asset prices under perfect matching information (i) increase in $\bar{\delta}$, the level of matching profitability; (ii) decrease in $\sigma_\delta^2$, the risk of the relationship; and (iii) increase in $\tilde{\rho}$, the level of matching intensity, if and only if $\tilde{\rho} < \frac{\bar{\delta}}{\gamma\sigma_\delta^2}$.*

Corollary 1 is intuitive. Prices will increase with the expected benefit of the relationship due to its positive impact on the cash flow, decrease with the underlying risk of the relationship due to its negative impact on the variance of the cash flow. Higher matching intensity increases both the mean and the variance of the underlying cash flow and so it increases asset prices only when the mean effect is sufficiently large. Since the mean effect increases linearly with $\tilde{\rho}$ but the variance effect increases quadratically with $\tilde{\rho}$ (and thus increases faster), higher matching intensity increases asset prices only when $\tilde{\rho}$ is sufficiently small.

### 3.3 Asset Prices under Matching Uncertainty

Facing uncertainty about matching intensity, the representative investor has to compute her expected utility without knowing $\tilde{\rho}$. For a given realization of $\tilde{\rho}$, final wealth follows a normal distribution. If $\tilde{\rho} = \rho_h$, then $\tilde{W} \sim N(\mu_W(h), \sigma_W^2(h))$, while if $\tilde{\rho} = \rho_l$, then $\tilde{W} \sim N(\mu_W(l), \sigma_W^2(l))$, where $\mu_W(s)$ and $\sigma_W^2(s)$ for $s \in \{l, h\}$ are the expectation and variance of final wealth conditional on $\tilde{\rho} = \rho_s$, and are given by

$$\mu_W(s) = E[\tilde{W} \mid \tilde{\rho} = \rho_s] = W_0 R + q_A(\bar{V} + \rho_s \bar{\delta} - RP_A) + q_B(\bar{V} + \rho_s \bar{\delta} - RP_B),$$

and

$$\sigma_W^2(s) = V[\tilde{W} \mid \tilde{\rho} = \rho_s] = q_A^2(\sigma_V^2 + \rho_s^2 \sigma_\delta^2) + q_B^2(\sigma_V^2 + \rho_s^2 \sigma_\delta^2) + 2 q_A q_B \rho_s^2 \sigma_\delta^2.$$

However, before $\tilde{\rho}$ is realized, final wealth does not follow a normal distribution and we cannot apply standard results of the CARA-Normal framework. The distribution of final wealth is a mixture of two normal distributions. To solve the portfolio choice, we need to calculate the expected utility of final wealth by using the Law of Iterated Expectations, resulting in:

$$\begin{aligned} EU &= E\left[E(-\exp(-\gamma \tilde{W}) \mid \tilde{\rho})\right] \\ &= -\pi \exp\left[-\gamma\left(\mu_W(h) - \frac{\gamma \sigma_W^2(h)}{2}\right)\right] - (1-\pi)\exp\left[-\gamma\left(\mu_W(l) - \frac{\gamma \sigma_W^2(l)}{2}\right)\right]. \end{aligned} \quad (5)$$

If we take the first-order conditions with respect to $q_A$ and $q_B$ and plug in the market-clearing conditions $q_A = 1/2$ and $q_B = 1/2$, we can compute prices for asset $j \in \{A, B\}$ as specified in the following proposition:

**Proposition 1** (Asset Prices under Matching Uncertainty). *The price of asset* $j \in \{A, B\}$ *is given* *by*

$$\begin{aligned} P_j &= \frac{\bar{V} - \frac{\gamma}{2}\sigma_V^2}{R} \\ &+ \frac{\bar{\delta}}{R}\left[\frac{\pi\rho_h e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2 - \gamma\rho_h\bar{\delta}} + (1-\pi)\rho_l e^{\frac{1}{2}\gamma^2\rho_l^2\sigma_\delta^2 - \gamma\rho_l\bar{\delta}}}{\pi e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2 - \gamma\rho_h\bar{\delta}} + (1-\pi) e^{\frac{1}{2}\gamma^2\rho_l^2\sigma_\delta^2 - \gamma\rho_l\bar{\delta}}}\right] \\ &- \frac{\gamma\sigma_\delta^2}{R}\left[\frac{\pi\rho_h^2 e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2 - \gamma\rho_h\bar{\delta}} + (1-\pi)\rho_l^2 e^{\frac{1}{2}\gamma^2\rho_l^2\sigma_\delta^2 - \gamma\rho_l\bar{\delta}}}{\pi e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2 - \gamma\rho_h\bar{\delta}} + (1-\pi) e^{\frac{1}{2}\gamma^2\rho_l^2\sigma_\delta^2 - \gamma\rho_l\bar{\delta}}}\right]. \end{aligned} \tag{6}$$

The asset price in this economy has three distinctive terms. The first term of the price would be the price of the asset if there was no common element $\tilde{\Delta}$ in the cash flows, that is, firms $A$ and $B$ would be independent of each other. The second term captures the benefit of having a relationship, while the third term, instead, captures the cost of forming a relationship. Note that both the benefit and the cost term come from expectations of utilities taken with respect to the prior distribution of $\tilde{\rho}$, hence containing the exponential terms. In other words, we can rewrite equation (6), analogously to equation (4), as

$$P_j = \frac{\widetilde{V} - \frac{\gamma}{2}\sigma_V^2}{R} + \frac{\bar{\delta}}{R}E^Q[\widetilde{\rho}] - \frac{\gamma\sigma_\delta^2}{R}E^Q[\widetilde{\rho}^2], \tag{7}$$

where

$$E^Q[\widetilde{\rho}^j] = \pi^{ra}\rho_h + (1-\pi^{ra})\rho_l,$$

and

$$\pi^{ra} = \frac{\pi e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2 - \gamma\rho_h\bar{\delta}}}{\pi e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2 - \gamma\rho_h\bar{\delta}} + (1-\pi) e^{\frac{1}{2}\gamma^2\rho_l^2\sigma_\delta^2 - \gamma\rho_l\bar{\delta}}}. \tag{8}$$

can be viewed as the risk-adjusted prior probability of $\tilde{\rho} = \rho_h$, for an investor with CARA utility and a portfolio of firms with correlated cash flows, and $Q$ is the risk-adjusted probability measure where $\rho = \rho_H$ with probability $\pi^{ra}$.

Note that when $\rho_h = \rho_l = \rho$, i.e., when there is no relationship uncertainty, we have the standard result in a CARA-Normal setup that the price of the relationship part is equal to the mean of cash flow minus the product of twice the risk-aversion coefficient (as there are two firms) and the variance of cash flow (discounted by the risk-free rate), i.e., the price is only determined by the first and second moments of cash flow,

$$\begin{aligned} P_j &= \frac{\bar{V} - \frac{\gamma}{2}\sigma_V^2}{R} + \frac{\bar{\delta}\rho}{R} - \frac{\gamma\sigma_\delta^2\rho^2}{R} \\ &= \frac{\bar{V} - \frac{\gamma}{2}\sigma_V^2}{R} + \frac{E[\tilde{\delta}\rho]}{R} - \frac{\gamma var[\tilde{\delta}\rho]}{R}. \end{aligned}$$

When $\rho_h \neq \rho_l$, i.e., in the presence of relationship uncertainty, the price of the relationship part is determined by the higher order moments of cash flow. As discussed in Heinle et al. (2018), different levels of skewness (and more generally odd moments) in the cash flow distribution results in different allocations of risk in states where the investor has different levels of wealth and thus different levels of disutility from taking risk, whereas different levels of kurtosis (and more generally even moments) results in different probability of extreme outcomes that risk-averse investors view unfavorably. Specifically, we can write the price expression as

$$\begin{aligned} P_j &= \frac{\bar{V} - \frac{\gamma}{2}\sigma_V^2}{R} + \frac{E[\tilde{\delta}\tilde{\rho}]}{R} - \frac{\gamma var[\tilde{\delta}\tilde{\rho}]}{R} \\ &\quad + \frac{\bar{\delta}(\pi^{ra} - \pi)(\rho_h - \rho_l)}{R} - \frac{\gamma}{R}\{\sigma_\delta^2(\pi^{ra} - \pi)(\rho_h^2 - \rho_l^2) - \bar{\delta}^2\pi(1-\pi)(\rho_h - \rho_l)^2\} \\ &= \frac{\bar{V} - \frac{\gamma}{2}\sigma_V^2}{R} + \frac{\phi E[\tilde{\delta}\tilde{\rho}]}{R} - \frac{\gamma}{R}\zeta, \end{aligned} \tag{9}$$

where

$$E[\tilde{\delta}\tilde{\rho}] = \bar{\delta}[\rho_h\pi + \rho_l(1-\pi)],$$

$$var[\tilde{\delta}\tilde{\rho}] = \sigma_\delta^2[\pi\rho_h^2 + (1-\pi)\rho_l^2] + \bar{\delta}^2\pi(1-\pi)(\rho_h - \rho_l)^2,$$

$$\phi = 1 + \frac{(\pi^{ra} - \pi)(\rho_h - \rho_l)}{\rho_h\pi + \rho_l(1-\pi)},$$

and

$$\zeta = \sigma_\delta^2[\pi^{ra}\rho_h^2 + (1-\pi^{ra})\rho_l^2].$$

As one can clearly see in the second line of equation (9), in addition to the usual mean and variance term, $P_j$ is determined by the difference between $\pi^{ra}$ and $\pi$, which clearly depends on higher order moments, based on a Taylor series expansion argument of the exponential terms. In the third line of equation (9), we compare our price expression to that of equation (3) in Lemma 1 of Heinle et al. (2018). Note that different from that in Heinle et al. (2018), $\phi$ is not necessarily always bigger than 1 as it depends on the relative magnitude of $\pi^{ra}$ and $\pi$. We are able to show that $\pi^{ra} > \pi$, and thus $\phi > 1$ if and only if

$$\frac{2\bar{\delta}}{\gamma\sigma_\delta^2} < \rho_h + \rho_l.$$

In other words, one need $\frac{2\bar{\delta}}{\gamma\sigma_\delta^2}$ to be sufficiently low relative to $\rho$ for the prior mean to have an amplifying effect on prices. Intuitively, when $\bar{\delta}$ is sufficiently low or $\sigma_\delta^2$ is sufficiently high, investors face more downside risk of the factor relative to the factor exposure. Uncertainty about factor exposure therefore preserves the upside potential and amplifies the effect of prior mean on prices. Note that this difference stems from the fact that the distribution of $\tilde{\Delta}$ in our setting is not necessarily positively skewed but is positively skewed in Heinle et al. (2018) when the mean of both components of the cash flow is positive.

Also different from Heinle et al. (2018), we can show that it is not necessarily true that $\zeta > var[\tilde{\delta}\tilde{\rho}]$, that is, uncertainty about factor exposure also does not necessarily increase kurtosis, which we conjecture is because our factor exposure distribution is not symmetric around the mean, as is the case in Heinle et al. (2018). When the factor exposure distribution is symmetric around the mean, more uncertainty increases the probability of tail events occurring in both tails thus amplifying kurtosis, which is not necessarily true if factor exposure distribution is highly skewed to one tail. Please see the Appendix for more details of the statistical properties of $\tilde{\Delta}$.

In contrast to the case with perfect matching information, asset prices under matching uncertainty are generally ambiguous in $\bar{\delta}$ and $\sigma_\delta^2$ due to their ambiguous effects on higher order moments of the cash flow distribution so we cannot make any general statements. We now examine the firms' optimal disclosure strategies of relationships.

# 4 Voluntary Disclosure of Relationship Intensities

In this section, we study the optimal disclosure policy about matching intensity $\tilde{\rho}$. To capture relationship disclosure, we assume that firms can commit to disclosing information about their matching intensity $\tilde{\rho}$. Such assumption of ex-ante commitment of voluntarily disclosing information is quite common in the literature (e.g., Diamond (1985)) and is consistent with the findings of a large stream of empirical literature showing that firms can ex-ante commit to disclosing with various information quality by, e.g., providing forecasts with different frequency and quality (e.g.,, Ajinkya et al. (2005), Karamanou and Vafeas (2005) and Baginski and Rakow (2012)). In principle, we can also model relationship disclosure as disclosing information about $\tilde{\delta}$ or about $\tilde{\Delta}$. We do not explore these alternatives in the current paper.

Before the realization of $\tilde{\rho}$, firms commit to a joint disclosure policy that will take place once firms observe the realization of $\tilde{\rho}$. They will send a message $\tilde{m} \in \{h, l\}$ to indicate if they are in the state of $\rho_h$ or $\rho_l$. Specifically, firms can choose the following probability $\alpha \in [\frac{1}{2}, 1]$ at zero cost:

$$\begin{aligned} Pr(\tilde{m} = h \mid \tilde{\rho} = \rho_h) &= Pr(\tilde{m} = l \mid \tilde{\rho} = \rho_l) = \alpha, \\ Pr(\tilde{m} = l \mid \tilde{\rho} = \rho_h) &= Pr(\tilde{m} = h \mid \tilde{\rho} = \rho_l) = 1 - \alpha. \end{aligned} \tag{10}$$

In the limit, when $\alpha = 1$, firms provide perfect disclosure of the realization of $\tilde{\rho}$. Instead, when $\alpha = 1/2$, firms provide no information about the realization of $\tilde{\rho}$. Any $\alpha$ in between will generate only partial disclosure about the realization of $\tilde{\rho}$.

We augment the main conceptual framework to a setup with three stages. The timeline of the augmented setup is described by Figure 1. At $t = 1$, before the realization of $\tilde{\rho}$, firms can commit to a disclosure policy $\alpha$. At $t = 2$, firms observe $\tilde{\rho}$ and provide a joint message $\tilde{m} \in \{h, l\}$ based on the disclosure policy $\alpha$. At $t = 3$, the investor updates her beliefs about the realization of $\tilde{\rho}$ based on the message received and the disclosure policy, decides her portfolio choice, and asset prices are determined in equilibrium.

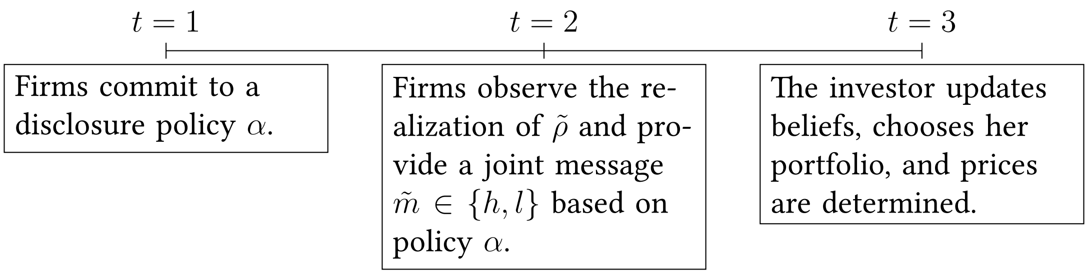



Figure 1: Timeline with Voluntary Disclosure of Relationships

Firms will choose the disclosure policy $\alpha$ that minimizes their costs of capital (CoC henceforth) given by $CoC_j \equiv E[\tilde{F}_j - P_j]$ for $j = \{A, B\}$. Since the decision on disclosure is made before firms observe the realization of $\tilde{\rho}$, minimizing the costs of capital is equivalent to maximizing the expected price in this setting without real effects (i.e., without endogenous choice of forming relationship). Specifically, since $E[\tilde{F}_j]$ is a constant independent of $\alpha$, then $\min_\alpha E[\tilde{F}_j - P_j]$ is equivalent to $\max_\alpha E[P_j]$. The model is solved using backward induction. First, we solve for the investor's demand functions and asset prices. Second, we solve for the firms' committed disclosure policies that maximize the firms' expected prices.

The investor's problem is the same as that derived in Section 3 for the conceptual framework with probabilities $\pi$ and $1-\pi$ being updated based on the message received at $t = 2$. Let us denote $\pi^h \equiv Pr(\tilde{\rho} = \rho_h \mid \tilde{m} = h)$ and $\pi^l \equiv Pr(\tilde{\rho} = \rho_h \mid \tilde{m} = l)$ as the probabilities of $\tilde{\rho} = \rho_h$ that are assigned by the investor after receiving the message $\tilde{m} = h$ and $\tilde{m} = l$, respectively. Accordingly, $1 - \pi^h$ and $1 - \pi^l$ are the posterior probabilities of $\tilde{\rho} = \rho_l$ assigned by the investor after receiving the message $m = h$ and $m = l$, respectively. Following Bayes' rule, these probabilities are given by

$$\begin{aligned} \pi^h = Pr(\tilde{\rho} = \rho_h \mid \tilde{m} = h) &= \frac{Pr(\tilde{m} = h \mid \tilde{\rho} = \rho_h) Pr(\tilde{\rho} = \rho_h)}{Pr(\tilde{m} = h)} = \frac{\alpha\pi}{\alpha\pi + (1-\alpha)(1-\pi)}, \\ \pi^l = Pr(\tilde{\rho} = \rho_h \mid \tilde{m} = l) &= \frac{Pr(\tilde{m} = l \mid \tilde{\rho} = \rho_h) Pr(\tilde{\rho} = \rho_h)}{Pr(\tilde{m} = l)} = \frac{(1-\alpha)\pi}{(1-\alpha)\pi + \alpha(1-\pi)}. \end{aligned} \tag{11}$$

There will be one asset price for $\tilde{m} = h$ and another one for $\tilde{m} = l$. The expression of the prices will be similar to the one in the conceptual framework in equation (6), but with adjusted probabilities $\pi^m$ for $\tilde{m} \in \{h, l\}$. Hence, for any message $\tilde{m} \in \{h, l\}$, the price $P_j(\alpha; \tilde{m})$ for $j \in \{A, B\}$ is given by

$$\begin{aligned} P_j(\alpha;\tilde{m}) &= \frac{\bar{V} - \frac{\gamma}{2}\sigma_V^2}{R} + \\ &+ \frac{\bar{\delta}}{R}\left[\frac{\pi^m \rho_h e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2 - \gamma\rho_h\bar{\delta}} + (1-\pi^m)\rho_l e^{\frac{1}{2}\gamma^2\rho_l^2\sigma_\delta^2 - \gamma\rho_l\bar{\delta}}}{\pi^m e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2 - \gamma\rho_h\bar{\delta}} + (1-\pi^m) e^{\frac{1}{2}\gamma^2\rho_l^2\sigma_\delta^2 - \gamma\rho_l\bar{\delta}}}\right] \\ &- \frac{\gamma\sigma_\delta^2}{R}\left[\frac{\pi^m \rho_h^2 e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2 - \gamma\rho_h\bar{\delta}} + (1-\pi^m)\rho_l^2 e^{\frac{1}{2}\gamma^2\rho_l^2\sigma_\delta^2 - \gamma\rho_l\bar{\delta}}}{\pi^m e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2 - \gamma\rho_h\bar{\delta}} + (1-\pi^m) e^{\frac{1}{2}\gamma^2\rho_l^2\sigma_\delta^2 - \gamma\rho_l\bar{\delta}}}\right], \end{aligned} \tag{12}$$

where $\pi^m$ is the posterior probability after receiving message $m$ given by equation (11). Using the similar risk-adjusted probability formulation as in equation (7), we can rewrite the price expressions given disclosure as

$$P_j(\alpha;\tilde{m}) = \frac{\bar{V} - \frac{\gamma}{2}\sigma_V^2}{R} + \frac{\bar{\delta}}{R}[\rho_h \pi^{ram} + \rho_l(1-\pi^{ram})] - \frac{\gamma\sigma_\delta^2}{R}[\rho_h^2\pi^{ram} + \rho_l^2(1-\pi^{ram})], \tag{13}$$

where

$$\pi^{ram} = \frac{\pi^m e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2 - \gamma\rho_h\bar{\delta}}}{\pi^m e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2 - \gamma\rho_h\bar{\delta}} + (1-\pi^m) e^{\frac{1}{2}\gamma^2\rho_l^2\sigma_\delta^2 - \gamma\rho_l\bar{\delta}}} \tag{14}$$

is the risk-adjusted probability of $\tilde{\rho} = \rho_h$ given disclose of $m$.

Given the asset prices for any $\alpha$, firms will choose the joint optimal disclosure policy $\alpha^*$ that maximizes their expected asset prices. Since the two firms are symmetric, we can focus on the problem of one firm. The expected asset prices are given by

$$E[P_j(\alpha;\tilde{m})] = P_j(\alpha;\tilde{m}=h)Pr(\tilde{m}=h) + P_j(\alpha;\tilde{m}=l)Pr(\tilde{m}=l). \tag{15}$$

The next corollary shows that an ex-post disclosure of higher relationship intensity is not necessarily good news to the market, relative to the ex-ante price.

**Corollary 2.** *Disclosure of $\tilde{m} = h$ ($\tilde{m} = l$) increases (decreases) prices (relative to ex-ante prices) if and only if $\frac{\bar{\delta}}{\gamma\sigma_\delta^2} > (\rho_h + \rho_l)$.*

Corollary 2 is similar to Proposition 1 of Heinle et al. (2018) in the sense that disclosure of high relationship intensity is not necessarily good news to the market as the price may drop. High relationship intensity increases the expected cash flow but also increases the underlying risk of the expected cash flow. Therefore, high relationship intensity is good news if and only if the benefit of the increase in expected cash flow dominates the underlying risk. Since $\tilde{m}$ is binary, the Bayesian updating is not linear. Therefore we are not able to follow Heinle et al. (2018) to express the price as a simple expression. Due to this complication, we are not able to provide analytical results on the comparative statics of both the ex-ante (i.e., before any disclosure) and ex-post (i.e., after disclosure) prices. We provide some graphical illustrations instead. Figure 2 shows how the ex-ante price and the two ex-post prices (i.e., when $\tilde{m} = h$ and $\tilde{m} = l$) change with respect to $\bar{\delta}$ and $\sigma_\delta^2$, respectively, when disclosure quality (i.e., $\alpha$) varies. While the general trend is ambiguous, price increases with $\bar{\delta}$ for large values of $\bar{\delta}$ and decreases with $\sigma_\delta^2$ for large values of $\sigma_\delta^2$. Perhaps surprisingly, price can increase with $\sigma_\delta^2$ when $\sigma_\delta^2$ is not large. As will be discussed below, higher $\sigma_\delta^2$ increases both skewness and kurtosis of the undelying cash flow distribution. The former increases stock prices and the latter decreases prices for an investor with CARA utility. When $\sigma_\delta^2$ is not large, the skewness effect dominates, resulting in price increasing in $\sigma_\delta^2$. The parameter values are reported on top of the panels.

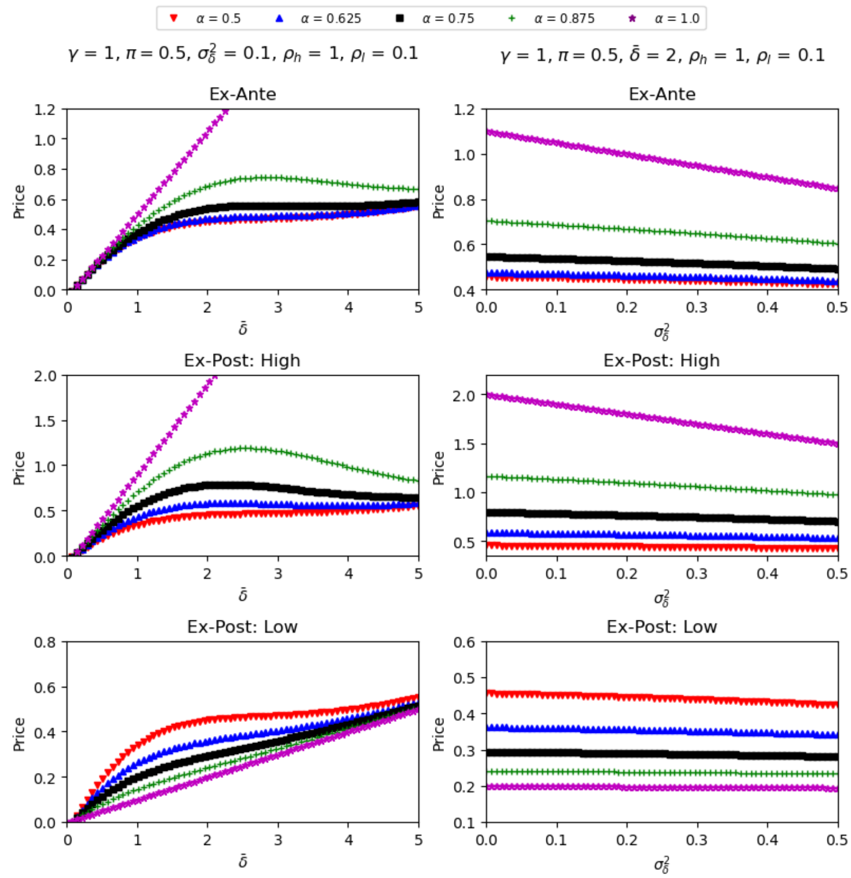
Given our focus is on the optimal disclosure policy, we now proceed to characterize it. Without loss of generality, we assume that if a firm is indifferent between any policy $\alpha$, it will choose full disclosure ($\alpha = 1$). The following proposition shows under which conditions firms will choose to opt for a full disclosure policy or a non-disclosure policy.[5]

**Proposition 2.** *The optimal disclosure policy $\alpha = \alpha^*$ is a corner solution and given by*

$$\alpha^* = \begin{cases} \frac{1}{2}, & \textit{if } \frac{\rho_h+\rho_l}{2} < \frac{\bar{\delta}}{\gamma\sigma_\delta^2} < \rho_h + \rho_l, \\ 1, & \textit{otherwise}\ . \end{cases} \tag{16}$$

[5] Note that while the proposition is about relationship intensity disclosure, in the special case of $\rho_l = 0$, the proposition is then about relationship disclosure.

Figure 2: Ex-Ante and Ex-Post Prices




Firms choose to commit to a non-disclosure policy $\alpha^* = \frac{1}{2}$ if and only if $\frac{\rho_h+\rho_l}{2} < \frac{\bar{\delta}}{\gamma\sigma_\delta^2} < \rho_h+\rho_l$. Otherwise, they commit to a full disclosure policy $\alpha^* = 1$. While the optimal disclosure policy is a corner solution, it is not robust to slight perturbations of the model. For example, we can add a disclosure cost that increases with $\alpha$. In this case, the optimal solution will still be no disclosure if and only if $\frac{\rho_h+\rho_l}{2} < \frac{\bar{\delta}}{\gamma\sigma_\delta^2} < \rho_h + \rho_l$ but may be interior otherwise. We choose not to include a disclosure cost in our main setting as we are essentially comparing $\alpha^* = 1$ with $\alpha^* = \frac{1}{2}$, which can be interpreted as more or less disclosure, while abstracting away from other factors that may affect disclosure precision. The conditions on $\alpha^* = 1$ versus $\alpha^* = \frac{1}{2}$ being the equilibrium can be used to derive predictions for when more disclosure is more likely, which we will discuss in more detail in Section 9 regarding empirical implications.

To further understand the conditions in Proposition 2, we rewrite equation (15) using the risk-adjusted probabilities as follows:

$$\begin{aligned}
& E[P_j(\alpha;\tilde{m})] \\
= \;& P_j(\alpha;\tilde{m}=h)\Pr(\tilde{m}=h) + P_j(\alpha;\tilde{m}=l)\Pr(\tilde{m}=l) \\
= \;& \{\frac{\bar{V}-\frac{\gamma}{2}\sigma_V^2}{R} + \frac{\bar{\delta}}{R}[\rho_h\pi^{rah}+\rho_l(1-\pi^{rah})] - \frac{\gamma\sigma_\delta^2}{R}[\rho_h^2\pi^{rah}+\rho_l^2(1-\pi^{rah})]\}\Pr(h) \\
& +\{\frac{\bar{V}-\frac{\gamma}{2}\sigma_V^2}{R} + \frac{\bar{\delta}}{R}[\rho_h\pi^{ral}+\rho_l(1-\pi^{ral})] - \frac{\gamma\sigma_\delta^2}{R}[\rho_h^2\pi^{ral}+\rho_l^2(1-\pi^{ral})]\}\Pr(l) \\
= \;& \frac{\bar{V}-\frac{\gamma}{2}\sigma_V^2}{R} + \frac{\phi E[\tilde{\delta}\tilde{\rho}]}{R} - \frac{\gamma}{R}\zeta, \qquad (17)
\end{aligned}$$

where

$$\phi = 1 + \frac{(\pi^{rad}-\pi)(\rho_h-\rho_l)}{\rho_h\pi+\rho_l(1-\pi)},$$

and

$$\zeta = \sigma_\delta^2[\pi^{rad}\rho_h^2 + (1-\pi^{rad})\rho_l^2],$$

where

$$\pi^{rad} = \pi^{rah}\Pr(h) + \pi^{ral}\Pr(l),$$

and we replace $\Pr(\tilde{m}=i)$ with $\Pr(i)$ for $i \in \{h,l\}$ to shorten the notation.

As in the analysis in Heinle et al. (2018), the marginal impact of a change in disclosure precision, $\alpha$, on price is

$$\begin{aligned}
\frac{\partial E[P_j(\alpha;\tilde{m})]}{\partial\alpha} &= \frac{1}{R}(E[\tilde{\delta}\tilde{\rho}]\frac{\partial\phi}{\partial\alpha} - \gamma\frac{\partial\zeta}{\partial\alpha}) \\
&= \frac{1}{R}[\underbrace{\bar{\delta}(\rho_h-\rho_l)\frac{\partial\pi^{rad}}{\partial\alpha}}_{\text{Skewness term}} - \underbrace{\gamma\sigma_\delta^2(\rho_h^2-\rho_l^2)\frac{\partial\pi^{rad}}{\partial\alpha}}_{\text{variance/kurtosis term}}]. \qquad (18)
\end{aligned}$$

We find that the sign of $\frac{\partial\pi^{rad}}{\partial\alpha}$ is ambiguous. So, unlike the case in Heinle et al. (2018), in our setting more precise disclosure does not unambiguously increase the skewness (or more generally, odd moment terms as discussed in Heinle et al. (2018)) or variance/kurtosis terms (or more generally, even moment terms as discussed in Heinle et al. (2018)). However, more precise disclosure always changes the skewness or variance/kurtosis terms in the same direction, therefore resulting in a tradeoff. We conjecture that the difference is due to that in Heinle et al. (2018), the distribution of factor exposure is normal and therefore symmetric with respect to the mean whereas in our setting, the factor exposure is binary and can only be positive, resulting in a truncated and therefore asymmetrical distribution with respect to the mean. In fact, we can show that in the hypothetical case of $\pi = \frac{1}{2}$ and $\rho_h = \rho = -\rho_l > 0$ (i.e., symmetric with respect to the mean), we can sign $\frac{\partial \pi^{rad}}{\partial \alpha}$, more specifically, $\frac{\partial \pi^{rad}}{\partial \alpha} \geq 0$ with the equality reached when $\alpha = \frac{1}{2}$, which will be consistent with the result in Heinle et al. (2018) that more precise disclosure increases skewness but decreases variance/kurtosis.

When $\frac{\partial \pi^{rad}}{\partial \alpha} > 0$, more precise disclosure increases the skewness and the marginal impact increases with $\bar{\delta}$ and $\rho_h - \rho_l$. Intuitively, uncertainty over the relationship creates skewness, because such uncertainty creates positive correlation in the mean and variance of the distribution. As $\bar{\delta}$ and $\rho_h - \rho_l$ increases, this uncertainty increases, resulting in a larger marginal impact of disclosure on skewness. When $\frac{\partial \pi^{rad}}{\partial \alpha} > 0$, more precise disclosure also increases the variance/kurtosis term and the marginal impact increases with $\sigma_\delta^2$ and $\rho_h^2 - \rho_l^2$. Intuitively, the larger the variation in $\tilde{\delta}$ and $\tilde{\rho}$, the larger the probability of tail risk, and thus the larger the impact of the variance/kurtosis term. Therefore, when $\bar{\delta}(\rho_h - \rho_l) > \gamma \sigma_\delta^2 (\rho_h^2 - \rho_l^2)$, or, equivalently, $\frac{\bar{\delta}}{\gamma \sigma_\delta^2} > (\rho_h + \rho_l)$, the impact of the skewness term dominates that of the variance/kurtosis term.

We can show that $\frac{\partial \pi^{rad}}{\partial \alpha} > 0$ if and only if $e^{\frac{1}{2}\gamma^2 \rho_l^2 \sigma_\delta^2 - \gamma \rho_l \bar{\delta}} > e^{\frac{1}{2}\gamma^2 \rho_h^2 \sigma_\delta^2 - \gamma \rho_h \bar{\delta}}$, or, equivalently, $\frac{\bar{\delta}}{\gamma \sigma_\delta^2} > \frac{1}{2}(\rho_h + \rho_l)$. To understand the intuition regarding the comparative statics of $\pi^{rad}$ with respect to $\alpha$, denote $k = \frac{e^{\frac{1}{2}\gamma^2 \rho_h^2 \sigma_\delta^2 - \gamma \rho_h \bar{\delta}}}{e^{\frac{1}{2}\gamma^2 \rho_l^2 \sigma_\delta^2 - \gamma \rho_l \bar{\delta}}}$. We can then write

$$\pi^{ram} = \frac{\pi^m k}{\pi^m k + (1 - \pi^m)},$$

for $m \in \{h, l\}$. Therefore we can view $k$ as the risk-adjusted weight on $\pi^h$, relative to the risk-neutral weight when $k = 1$, as in this case we can write

$$\pi^m = \frac{\pi^m}{\pi^m + (1 - \pi^m)}.$$

We can now express $\pi^{rad}$ as

$$\begin{aligned}
\pi^{rad} &= \pi^{rah} \Pr(h) + \pi^{ral} \Pr(l) \\
&= \frac{\pi^h k}{\pi^h k + (1 - \pi^h)} \frac{\alpha \pi}{\pi^h} + \frac{\pi^l k}{\pi^l k + (1 - \pi^l)} \frac{(1 - \alpha)\pi}{\pi^l} \\
&= \pi k [\frac{\alpha}{\pi^h k + (1 - \pi^h)} + \frac{(1 - \alpha)}{\pi^l k + (1 - \pi^l)}], \qquad (19)
\end{aligned}$$

where we used equation (11),

$$\Pr(h) = \alpha \pi + (1 - \alpha)(1 - \pi) = \frac{\alpha \pi}{\pi^h},$$

and

$$\Pr(l) = (1 - \alpha)\pi + \alpha(1 - \pi) = \frac{(1 - \alpha)\pi}{\pi^l}$$

,to arrive at the first equality in equation (19).

For the risk-neutral weight, $k = 1$ so $\pi^{rad} = \pi$ and thus is independent of $\alpha$.

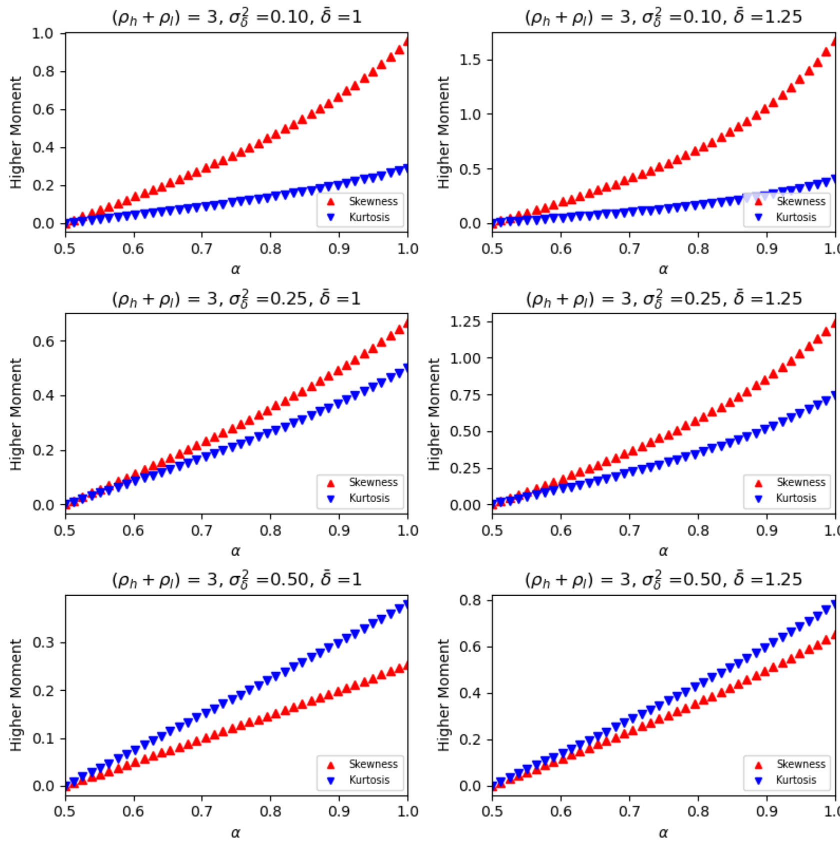
For the risk-adjusted case, note that $k < 1$ if and only if $e^{\frac{1}{2}\gamma^2 \rho_l^2 \sigma_\delta^2 - \gamma \rho_l \overline{\delta}} > e^{\frac{1}{2}\gamma^2 \rho_h^2 \sigma_\delta^2 - \gamma \rho_h \overline{\delta}}$, or, equivalently, $\overline{\delta} > \frac{\gamma \sigma_\delta^2}{2}(\rho_h + \rho_l)$. As discussed above, this inequality implies a larger variation in $\widetilde{\delta}$ and $\widetilde{\rho}$, a higher skewness and a larger tail risk, resulting in a lower risk-adjusted probability of $\widetilde{\rho} = \rho_h$ conditional on observing $\widetilde{m}$. In other words, $\pi^{rah} < \pi^h$ and $\pi^{ral} < \pi^l$. The sensitivity of $\pi^{ram}$ with respect to $\alpha$, meanwhile, becomes larger, as $\pi^{rah} \to 1$ and $\pi^{ral} \to 0$ as $\alpha \to 1$, regardless of $k$. In addition, while $\pi^{rah}$ increases in $\alpha$ and $\pi^{ral}$ decreases in $\alpha$, the first effect dominates as $|\frac{\partial \pi^h}{\partial \alpha}| > |\frac{\partial \pi^l}{\partial \alpha}|$. The reason is that since $\widetilde{m} = h$ is the matching signal, observing $\widetilde{m} = h$ is more informative of $\tilde{\rho} = \rho_h$ than observing $\widetilde{m} = l$, when disclosure becomes more precise. The intuition when $k > 1$ is analogous.

It is also worth mentioning that the condition when $\pi^{rad}$ decreases with $\alpha$, $\overline{\delta} < \frac{\gamma\sigma_\delta^2}{2}(\rho_h + \rho_l)$, is identical to the condition in the baseline model when $\pi^{ra} > \pi$. This implies that when there is excess weight on $\rho_h$ in price, this weight decreases when there is more disclosure. This feature, while nice and intuitive, is not coincidental, as both comes from the risk-adjusted weight on $\pi^h$ (relative to the risk neutral weight) $k$: When $k < 1$, a higher downside risk from $\widetilde{\rho} = \rho_l$ results in a higher risk-adjusted probability of $\widetilde{\rho} = \rho_h$ conditional on observing $\widetilde{m}$ and thus an excess weight on $\rho_h$; more disclosure, by revealing more information and alleviates such risk, reduces the excess weight.

In summary, when $\frac{\bar{\delta}}{\gamma\sigma_\delta^2} > \frac{1}{2}(\rho_h + \rho_l)$, $\frac{\partial\pi^{rad}}{\partial\alpha} > 0$ and more disclosure increases both the skewness term and the kurtosis term but the effect on the skewness term dominates, resulting in perfect disclosure maximizing the price; when $\frac{\bar{\delta}}{\gamma\sigma_\delta^2} < \frac{1}{2}(\rho_h + \rho_l)$, $\frac{\partial\pi^{rad}}{\partial\alpha} < 0$ and more disclosure decreases both the skewness term and the kurtosis term but the effect on the kurtosis term dominates (is more negative), again resulting in perfect disclosure maximizing the price; when $\frac{1}{2}(\rho_h + \rho_l) < \frac{\bar{\delta}}{\gamma\sigma_\delta^2} < \rho_h + \rho_l$, $\frac{\partial\pi^{rad}}{\partial\alpha} > 0$ and more disclosure increases both the skewness term and the kurtosis term but the effect on the kurtosis term dominates, resulting in no disclosure maximizing the price. Similar to Heinle et al. (2018), more disclosure does not necessarily decrease the cost of capital because its effect on the higher order moments terms is not unambiguously price-increasing. Different from Heinle et al. (2018), the asymmetric binary distribution in our setting results in more disclosure not having an unambiguously positive effect on skewness and negative effect on kurtosis. Perhaps more importantly, the binary structure provides us the tractability to be able to characterize the necessary and sufficient conditions for disclosure to increase the cost of capital, which is useful in our subsequent analysis regarding when firms choose to form relationship in the first place. As shown in Figure 3 and 4, more disclosure increases skewness and kurtosis when $\frac{\bar{\delta}}{\gamma\sigma_\delta^2} > \frac{1}{2}(\rho_h + \rho_l)$, whereas more disclosure reduces skewness and kurtosis when the reverse inequality holds.

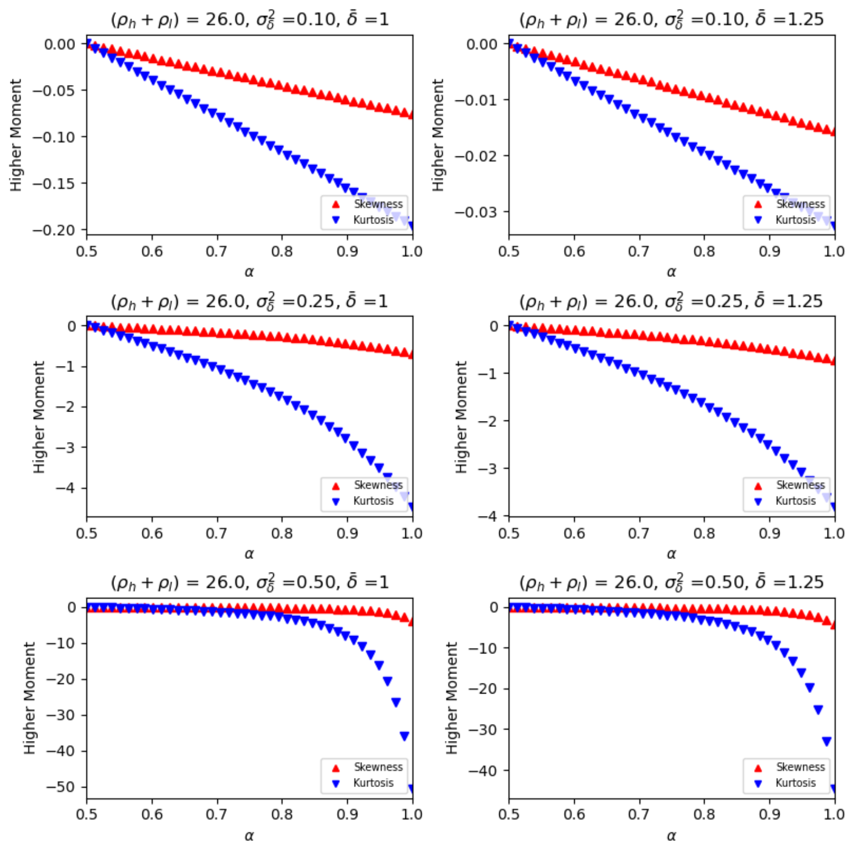
From a general intuition perspective, whether or not disclosure is optimal depends on the magnitude of $\overline{\delta}(\rho_h - \rho_l)$ relative to $\gamma\sigma_\delta^2(\rho_h^2 - \rho_l^2)$, which determines the dominance of skewness

Figure 3: Higher order moments increase with disclosure




versus variance/kurtosis. When $\overline{\delta}(\rho_h - \rho_l)$ is very large relative to $\gamma\sigma_\delta^2(\rho_h^2 - \rho_l^2)$, more precise disclosure, by informing the realization of $\widetilde{\rho}$, exacerbates the asymmetry of the distribution thus increasing the skewness. In addition, more precise disclosure increases the variance/kurtosis term, as the high value of $\overline{\delta}$ exacerbates the tail risk when $\tilde{\rho} = \rho_l$. However, the effect on the skewness term dominates, resulting in perfect disclosure being optimal. When $\overline{\delta}(\rho_h - \rho_l)$ is in the middle relative to $\gamma\sigma_\delta^2(\rho_h^2 - \rho_l^2)$, more precise disclosure still increases both the skewness and variance/kurtosis terms but the effect on the variance/kurtosis term dominates, resulting in no disclosure being optimal. When $\overline{\delta}(\rho_h - \rho_l)$ is very small relative to $\gamma\sigma_\delta^2(\rho_h^2 - \rho_l^2)$, the distribution of future cash flows exhibits relatively high tail risk, resulting in investors putting a higher risk-adjusted (relative to risk-neutral) probability of $\widetilde{\rho} = \rho_h$ conditional on disclosure. More precise disclosure not only decreases the variance/kurtosis term but also reduces the excess weight on $\widetilde{\rho} = \rho_h$, thus reducing skewness. However, the effect on the variance/kurtosis term dominates,

Figure 4: Higher order moments decrease with disclosure




resulting in perfect disclosure still being optimal.

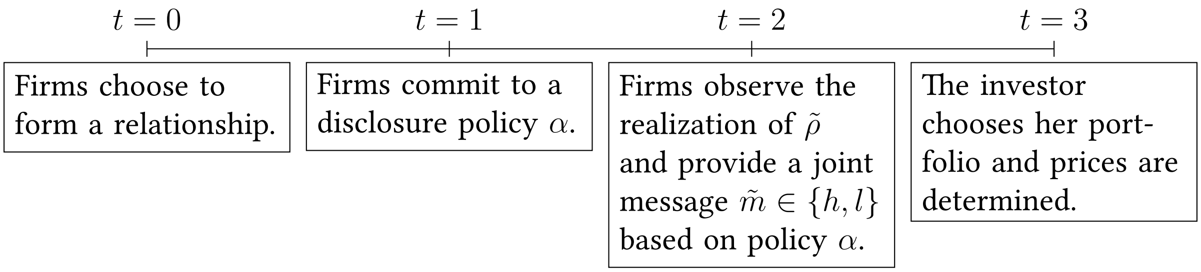
**Corollary 3.** *The price of asset $j \in \{A, B\}$ when the optimal policy is non-disclosure ($\alpha^* = \frac{1}{2}$) is given by equation (6). Instead, when the optimal policy is full disclosure ($\alpha^* = 1$), asset prices are given by equation (4) and the expected asset price is given by*

$$E[P_j(\alpha^* = 1; \tilde{m})] = \frac{\bar{V} - \frac{\gamma}{2}\sigma_V^2}{R} + \frac{\bar{\delta}}{R}(\pi\rho_h + (1-\pi)\rho_l) - \frac{\gamma\sigma_\delta^2}{R}(\pi\rho_h^2 + (1-\pi)\rho_l^2). \qquad (20)$$

## 5 Relationship Formation

In this section, we study the real effects of relationship disclosure, that is, how disclosing relationship affects firms' choices to form relationships in the first place. We augment the conceptual framework with disclosure from the previous section to a setting with four stages. The timeline

Figure 5: Timeline with Relationship Formation and Voluntary Relation Disclosure




of the economy is now given by Figure 5. At $t = 0$, firms choose whether to form a relationship. At $t = 1$, before the realization of $\tilde{\rho}$, firms can commit to a disclosure policy $\alpha$. At $t = 2$, firms observe $\tilde{\rho}$ and provide a joint message $\tilde{m} \in \{h, l\}$ based on the disclosure policy $\alpha$. At $t = 3$, the investor updates her beliefs about the realization of $\tilde{\rho}$ based on the message received and the disclosure policy, and decides her portfolio choice, and prices are determined in equilibrium.

As in the previous section, firms' objectives are to maximize expected prices. That is, a firm will choose to form a relationship if and only if the expected price of the firm when forming a relationship is higher than the expected price of the firm under no relationship, taking into account the strategic actions of the other firm. Without loss of generality, we assume that if a firm is indifferent between forming or not forming a relationship, the firm will choose not to form a relationship.

If firms choose not to have a relationship, then the cash flows of each firm have only one component and are given by $\tilde{F}_A = \tilde{V}_A$ and $\tilde{F}_B = \tilde{V}_B$, which corresponds to the degenerate case of $\pi = \rho_l = 0$ in Section 3. We use superscript "NR" to denote the case in which the two firms have not formed a relationship. Taking prices as given, demand for asset $j \in \{A, B\}$ is standard in a CARA-Normal framework and given by

$$q_j^{NR} = \frac{\bar{V} - RP_j^{NR}}{\gamma \sigma_V^2}. \tag{21}$$

The demand of the investor for asset $j$ is independent of the demand for the other asset in the economy. The demand depends positively on the expected excess returns and negatively on the variance of the asset and the risk aversion of the investor. We can compute prices using the market-clearing conditions: $q_A = \frac{1}{2}$ and $q_B = \frac{1}{2}$. The prices of the two firms are the same and given by

$$P_j^{NR} = \frac{\bar{V} - \frac{\gamma}{2}\sigma_V^2}{R}, \tag{22}$$

for $j \in \{A, B\}$. The price is the present discounted value of expected payoffs adjusted for the risk associated with holding the asset.

If firms choose to have a relationship, then cash flows have two components $\tilde{F}_A = \tilde{V}_A + \tilde{\Delta}$ and $\tilde{F}_B = \tilde{V}_B + \tilde{\Delta}$ as in the conceptual framework in Section 3. The second component $\tilde{\Delta}$ is given by equation (1) and the price for firm $j \in \{A, B\}$ is given by (12). Firms will compare the expected price (15) of forming a relationship under the optimal disclosure policy $\alpha^*$ from equation (16) with the price (22) under no relationship, taking into account the choice of the other firm. The payoff matrix is given by Figure 6 and the definition of the equilibrium of this game and the definitions of a relationship equilibrium, a no-relationship equilibrium, and a Pareto-dominant equilibrium are presented below.

Figure 6: Payoffs under Relationship Formation

| | | Firm $B$ | |
|---|---|---|---|
| | | Relation | No Relation |
| Firm $A$ | Relation | $E[P_A(\alpha^*;\tilde{m})], E[P_B(\alpha^*;\tilde{m})]$ | $P_A^{NR}, P_B^{NR}$ |
| | No Relation | $P_A^{NR}, P_B^{NR}$ | $P_A^{NR}, P_B^{NR}$ |

**Definition 1.** *An equilibrium consists of (i) prices when there is no relationship $P_A^{NR}$ and $P_B^{NR}$ that satisfy the market-clearing conditions and are given by (22); (ii) prices when there is a relationship $P_A(\alpha; m)$ and $P_B(\alpha, m)$ that satisfy the market-clearing conditions and are given by (12); (iii) a disclosure policy $\alpha$ that maximizes (15) and is given by (16); and (iv) the firms' decisions to form a relationship that form a Nash equilibrium in Pure Strategies of the game in Figure 6.*

**Definition 2.** *(i) A relationship equilibrium is an equilibrium in which both firms $A$ and $B$ choose to form a relationship. (ii) A no-relationship equilibrium is an equilibrium in which at least one of* *the firms $A$ or $B$ chooses not to form a relationship. (iii) An equilibrium is Pareto-dominant if it has the highest expected price across all equilibria.*

## 5.1 Equilibrium Characterization

The following Lemma shows that there always exists an equilibrium with no relationship.

**Lemma 1.** *A no-relationship equilibrium always exists.*

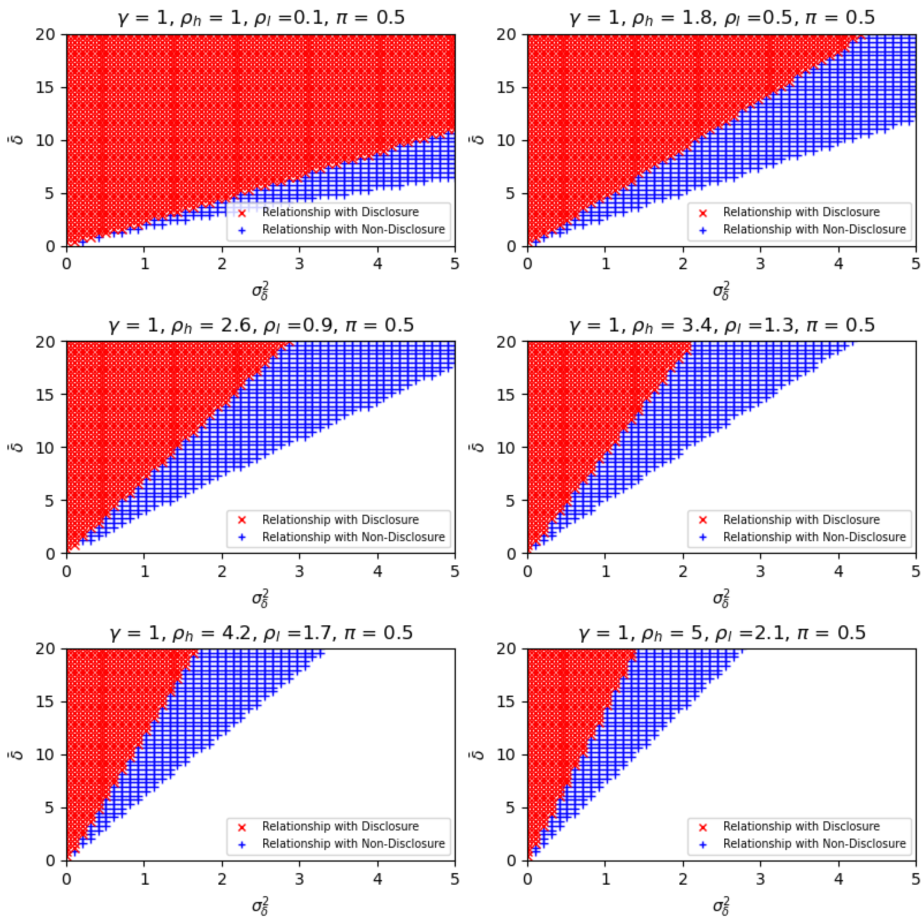
Intuitively, since a relationship is formed only when both firms choose to do so, no firm has any incentive to deviate from a no-relationship equilibrium as there is no benefit from a unilateral deviation. For the purpose of this paper, we will focus on the Pareto-dominant equilibrium. As mentioned above, a Pareto-dominant equilibrium is the equilibrium with the highest expected price.[6] There will be a Pareto-dominant equilibrium with a relationship when $E[P_j(\alpha^*; \tilde{m})] > P_j^{NR}$. The next proposition characterizes the conditions for the existence of a Pareto-dominant relationship equilibrium.

**Proposition 3.** *There exists a Pareto-dominant relationship equilibrium in which firms will form a relationship with full-disclosure ($\alpha^* = 1$) if and only if $\frac{\bar{\delta}}{\gamma\sigma_\delta^2}$ is sufficiently large. There exists a Pareto-dominant relationship equilibrium in which firms will form a relationship with non-disclosure ($\alpha^* = \frac{1}{2}$) if and only if $\frac{\bar{\delta}}{\gamma\sigma_\delta^2}$ is not too large and not too small (so that firms will find non-disclosure being optimal from Proposition 2).*[7]

Proposition 3 states that firms will form relationship if and only if $\frac{\bar{\delta}}{\gamma\sigma_\delta^2}$ is sufficiently large, that is, when the benefit of increased matching profitability from relationship formation is sufficiently larger than the cost of increased collaboration risk. Figure 7 shows the parameter range for which a Pareto-dominant relationship equilibrium exists. There are six parameters involved, that is, $\pi$, $\gamma$, $\rho_h$, $\rho_l$, $\bar{\delta}$, and $\sigma_\delta^2$. In the figure, we assume $\gamma = 1$ and $\pi = 0.5$, choose several values for $\rho_h$ and $\rho_l$, and show equilibria with relationship formation when we vary $\bar{\delta}$ and $\sigma_\delta^2$. We observe that

[6] In Section 8 we discuss the relation between Pareto-dominance and social welfare.
[7] The exact conditions are shown in the proof of Proposition 3 in the appendix.

Figure 7: Pareto-Dominant Equilibrium with Relationship Formation




equilibria with relationship formation can be Pareto-dominant under both a disclosure and a non-disclosure policy. Intuitively, when the matching profitability $\bar{\delta}$ of forming a relationship is high relative to the collaboration risk $\sigma_\delta^2$, a Pareto-dominant relationship equilibrium exists. When the matching profitability is very high relative to the collaboration risk of forming a relationship, firms choose to commit to disclosing the nature of their relationship, as the benefit of disclosure exceeds the cost. When the expected matching profitability is at intermediate levels relative to the collaboration risk of forming a relationship, firms prefer to commit to a non-disclosure regime, as the investor is (relatively) less concerned about the increase in diversification cost (relative to when the matching profitability is sufficiently low relative to the collaboration risk) and puts more weight on the scenario when $\rho$ is low in the absence of disclosure. Therefore, firms choose not to disclose. However, firms would still prefer to form a relationship as the expected benefit of forming a relationship still dominates the cost of increased diversification cost. Finally, we observe that when the expected matching profitability of forming relationship is low relative to the collaboration risk of increased diversification cost, there is a unique equilibrium where no relationship is formed.

## 5.2 Mandatory Disclosure

In this subsection, we study the implications of regulations mandating relationship disclosure, such as Regulation SFAS No. 131 under which firms must disclose their major operating segments and the existence of major customers. The next proposition shows that the introduction of mandatory disclosure ($\alpha = 1$) may lead to destruction of previously formed relationship under non-disclosure ($\alpha = \frac{1}{2}$).

**Proposition 4.** *Mandatory disclosure may destroy relationship formation. Specifically, for any firm* $j \in \{A, B\}$, $E[P_j(\alpha = \frac{1}{2}; \tilde{m})] > P_j^{NR} > E[P_j(\alpha = 1; \tilde{m})]$ *if and only if*

$$\max\left\{\frac{\rho_h + \rho_l}{2}, \frac{\pi\rho_h^2 e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2 - \gamma\rho_h\bar{\delta}} + (1-\pi)\rho_l^2 e^{\frac{1}{2}\gamma^2\rho_l^2\sigma_\delta^2 - \gamma\rho_l\bar{\delta}}}{\pi\rho_h e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2 - \gamma\rho_h\bar{\delta}} + (1-\pi)\rho_l e^{\frac{1}{2}\gamma^2\rho_l^2\sigma_\delta^2 - \gamma\rho_l\bar{\delta}}}\right\} < \frac{\bar{\delta}}{\gamma\sigma_\delta^2} < \frac{\pi\rho_h^2 + (1-\pi)\rho_l^2}{\pi\rho_h + (1-\pi)\rho_l},$$

*which requires* $\pi\rho_h > (1-\pi)\rho_l$. *Mandatory disclosure will not destroy relationship formation when* $\frac{\bar{\delta}}{\gamma\sigma_\delta^2} > \frac{\pi\rho_h^2 + (1-\pi)\rho_l^2}{\pi\rho_h + (1-\pi)\rho_l}$. *In particular, when* $\frac{\bar{\delta}}{\gamma\sigma_\delta^2} \in (\frac{\pi\rho_h^2 + (1-\pi)\rho_l^2}{\pi\rho_h + (1-\pi)\rho_l}, \rho_h + \rho_l)$, *mandatory disclosure will force firms to disclose but will keep relationship formation intact.*

When the parameter condition in the above proposition is satisfied, moving from a non-disclosure regime ($\alpha = \frac{1}{2}$) to a disclosure regime ($\alpha = 1$) would break relationships previously formed. Intuitively, when the benefit of disclosure relative to the cost is in the intermediate region, firms would optimally form a relationship but not disclose, because $E[P_j(\alpha = \frac{1}{2}; \tilde{m})] > P_j^{NR}$, as the benefit of forming a relationship is still sufficiently large. However, once regulation forcing disclosure is introduced, then forming relationships would be suboptimal because $P_j^{NR} > E[P_j(\alpha = 1; \tilde{m})]$, that is, forcing firms to disclose results in firms not establishing relationship in the first place.

Figure 8: Mandatory Disclosure and Destruction of Relationships

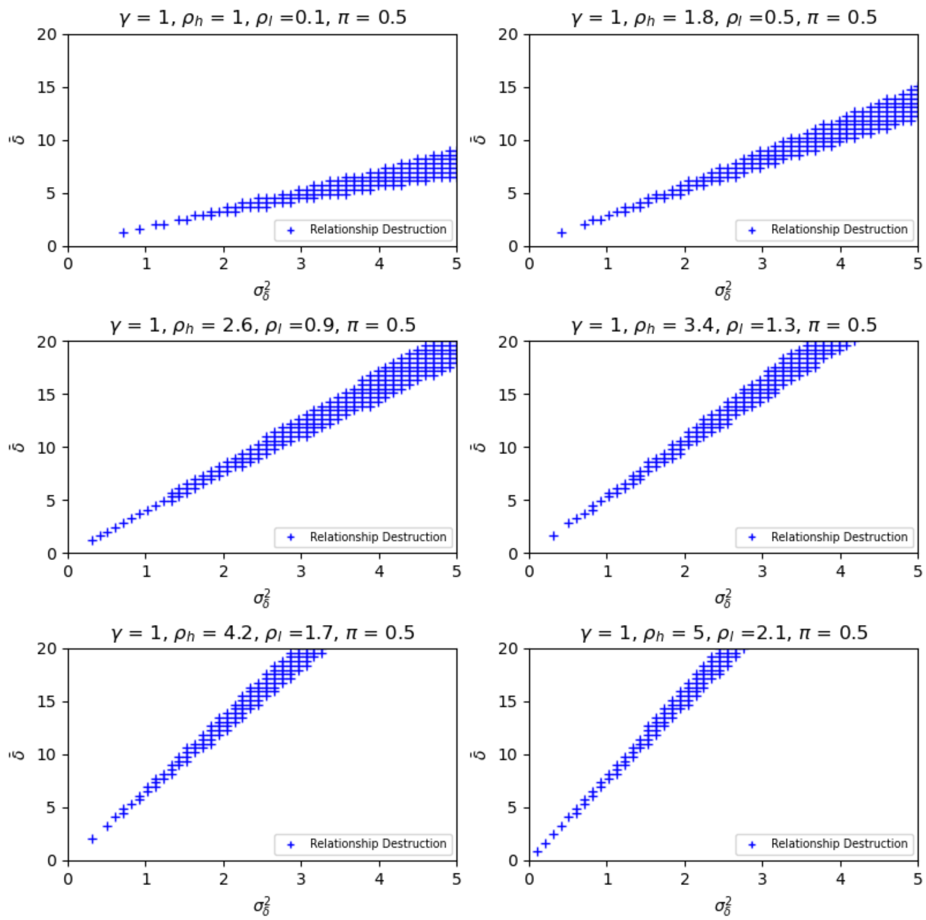



Figure 8 shows the parameter range for which forcing disclosure destroys the formation of relationships. In the figure, we assume $\gamma = 1$ and $\pi = 0.5$, choose several values for $\rho_h$ and $\rho_l$, and show relationship formation destruction when we vary parameters $\bar{\delta}$ and $\sigma_\delta^2$. For most combinations of $\rho_h$ and $\rho_l$, there exists a region for which relationships are formed only under the non-disclosure policy when $\bar{\delta}$ relative to $\sigma_\delta^2$ is in an intermediate range. Thus, when the matching profitability of forming a relationship is at some intermediate level relative to the collaboration risk, mandatory disclosure may destroy relationship formation and will affect the extensive margin of a relationship between two firms.

# 6 N-Firm Extension

In the main setting we focus on understanding the effects on asset prices and disclosure of uncertainty of a relationship in a two-firm setting. This naturally raises the concern regarding whether the results hold more generally in an N-firm setting. In this section we extend our analysis to such a setting: N firms with idiosyncratic relationships. We show analytically that in our setting with N firms, the relationship risk is still priced even in a large economy when $N \to \infty$. We also show numerically that our disclosure result, Proposition 2, holds qualitatively in the N-firm setting.

Specifically, we assume that firm $i$'s cash flow is characterized by

$$\tilde{F}_i = \tilde{V}_i + \sum_{j \neq i} \tilde{\rho}_{i,j} \tilde{\delta}_{i,j}, \tag{23}$$

where $\tilde{\delta}_{i,j} = \tilde{\delta}_{j,i}$ and $\tilde{\rho}_{i,j} = \tilde{\rho}_{j,i}$ for all $i \neq j$. As in the two-firm setting, the cash flow of each firm still has an idiosyncratic component. In addition, each relationship between firm $i$ and $j$ increases the cash flow of both firms in a symmetrical way. Final wealth is then given by $\tilde{W} = W_0 R + \sum_{i=1}^{N} q_i (\tilde{F}_i - RP_i)$.

As before we assume that $\tilde{V}_i \sim N(\bar{V}, \sigma_V^2)$, $\tilde{\delta}_{i,j} \sim N(\bar{\delta}, \sigma_\delta^2)$ and $\tilde{\rho}_{i,j}$ characterizes the matching intensity of firm $i$ with firm $j$ given by

$$\tilde{\rho}_{i,j} = \begin{cases} \rho_h & \text{with probability } \pi_{i,j}, \\ \rho_l & \text{with probability } 1 - \pi_{i,j}. \end{cases}$$

All random variables $\tilde{V}_i$, $\tilde{\rho}_{i,j}$, $\tilde{\delta}_{i,j}$ are mutually independent, and $\tilde{\rho}_{i,j}$ and $\tilde{\delta}_{i,j}$ are independent across firms, except that $\tilde{\rho}_{i,j} = \tilde{\rho}_{j,i}$ and $\tilde{\delta}_{i,j} = \tilde{\delta}_{j,i}$. Based on the realizations of the matching intensity for N firms, there are $S = 2^{N(N-1)/2}$ possible states of the economy. We use $s \in \{1, 2, ..., 2^{N(N-1)/2}\}$ to denote one of the possible states of the economy. The probability of each state $s$ occurring can be derived using the relationship matching intensity probabilities $\pi_{i,j}$ that lead to that specific state for each of the N fims (essentially the product of the matching intensity probabilities that leads to $s$). Denote $P(s)$ as the probability of state $s$ occurring. Denote $\rho_{i,j}(s)$ as the realized matching intensity between firm $i$ and $j$ in economy state $s$, and

$$\rho(s) = \begin{bmatrix} \rho_{1,2}(s) \\ \rho_{1,3}(s) \\ \dots \end{bmatrix}$$

as the vector containing the bilateral matching intensity among all firms in state $s$.

As in the baseline model, we assume that there is a representative investor with CARA preference. Each firm has $\frac{1}{N}$ shares in supply. Note that this specification is identical to specifying that each firm has 1 share in supply and that the number of investors increases in the same speed as the number of firms, since the market-clearing conditions under both specifications are essentially the same. Intuitively, both specifications serve to adjusting the risk-bearing capacity of the economy according to the number of firms.

To solve the portfolio choice and find the asset prices in the N-firm setting, we need to calculate the expected utility of final wealth, which is

$$EU = E\left[E(-\exp(-\gamma\tilde{W}) \mid s)\right] = -\sum_{s=1}^{S} P(s) \exp\left(-\gamma[\mu(s) - \frac{1}{2}\gamma\sigma^2(s)]\right), \tag{24}$$

where

$$\begin{aligned} \mu(s) &= E(\tilde{W} \mid s) = W_0 R + \sum_{i=1}^{N}\left(q_i(\bar{V} + \sum_{j\neq i}\rho_{i,j}(s)\bar{\delta} - RP_i)\right), \\ \sigma^2(s) &= Var(\tilde{W} \mid s) = \sum_{i=1}^{N} q_i^2\sigma_V^2 + \sigma_\delta^2 \sum_{i=1}^{N}\left(q_i^2\sum_{j\neq i}\rho_{i,j}(s)^2 + q_i\sum_{j\neq i} q_j\rho_{i,j}(s)^2\right). \end{aligned} \tag{25}$$

If we take the first-order conditions with respect to $q_i$ and plug in the market condition $q_i = \frac{1}{N}$ for all $i$, we can compute asset prices as specified in the following proposition:

**Proposition 5** (Asset Prices in the N-firm setting)**.** *The price of asset $i \in N$ is given by*

$$P_i = \frac{1}{R} \sum_{s=1}^{S} \frac{P(s) e^{\phi(s)}}{\sum_{k=1}^{S} P(k) e^{\phi(k)}} \left( \bar{V} + \bar{\delta} \sum_{j \neq i} \rho_{i,j}(s) - \frac{\gamma}{N} \left( \sigma_V^2 + 2\sigma_\delta^2 \sum_{j \neq i} \rho_{i,j}(s)^2 \right) \right), \tag{26}$$

*where*

$$e^{\phi(s)} = \exp\left( -\frac{\gamma \bar{\delta}}{N} \sum_{i=1}^{N} \sum_{j \neq i} \rho_{i,j}(s) + \frac{\gamma^2 \sigma_\delta^2}{N^2} \sum_{i=1}^{N} \sum_{j \neq i} \rho_{i,j}(s)^2 \right).$$

In a large economy, the price in (26) when $N \to \infty$ would explode unless we scale $\bar{\delta}$ by $\frac{1}{N}$, such that $\bar{\delta} = \frac{D}{N}$ for a constant $D$. Intuitively, as $N$ becomes large, each firm forms $N-1$ independent relations with other firms, leading to an explosion of the expected payoffs. The following corollary is striking as it challenges the conventional wisdom that idiosyncratic risk is usually diversified away in large economies.

**Corollary 4.** *In a large economy with idiosyncratic relationships, relationship risk does not vanish and does get priced in.*

Interestingly, investors are not able to diversify all the relationship risk away. Intuitively, as $N$ becomes large, each firm forms $N-1$ independent relations with other firms, and so each firm's risk increases exactly at the same speed as the number of firms. As a result, the risk of a representative investor's equilibrium portfolio remains a constant and does not vanish, and so relationship risk is priced in the limit. [8] All our results of the benchmark model will remain in a large economy as long as we scale $\bar{\delta}$ properly.

## 6.1 Optimal Disclosure

We now examine the firm's optimal disclosure in the N-firm setting. To facilitate discussions, we assume that firms $i$ and $j$ form a joint committee to make the disclosure decisions about the

[8] Statistically, this pricing feature in our model is similar to the way how Verrecchia (1982) models noisy trading with traders' asset endowments in an economy populated with a large number of traders. In his noisy-rational-expectations-equilibrium model, the risky asset endowment of each trader is mutually independent, but as the number of traders grows, the variance of each trader's endowment grows at the same speed as trader numbers, so that the risky endowment per capita – which serves as the noisy trading in Verrecchia (1982)'s setting – approaches a constant in the limit.

relationship between these two firms. The committee's objective function is to maximize the sum of the expected prices of the two firms. Specifically, before the realization of $\tilde{\rho}_{i,j}$, firms $i$ and $j$ form the joint committee to commit to a joint disclosure policy that will take place after observing the realization of $\tilde{\rho}_{i,j}$. The disclosure is still modelled as that firms send a message $\tilde{m}_{i,j} \in \{h, l\}$ to indicate if they are in a state with $\rho_h$ or $\rho_l$. The committee determines the probability $\alpha_{i,j} \in [\frac{1}{2}, 1]$ at zero cost as in equation (10), to maximize the sum of expected prices $P_i$ and $P_j$. We also assume that all bilateral committees make decisions simultaneously and symmetrically to preserve tractability. This implies that the committee between firm $i$ and firm $j$ perceive that all other relationship disclosure policies are equal to $\alpha^*$.

The timeline of the model follows the main setting. At $t = 1$, firms commit to a disclosure policy $\alpha_{i,j}$ for each bilateral relationship in the economy. At $t = 2$, firms observe the realization of $\tilde{\rho}_{i,j}$ and provide a joint message based on policy $\alpha_{i,j}$. Finally, at $t = 3$, the investor updates beliefs and chooses her portfolio which determines prices.

After receiving a message $\tilde{m}_{i,j}$, the investor will update individual relationship probabilities as follows:

$$\begin{aligned} \pi_{i,j}^h &= Pr(\tilde{\rho}_{i,j} = \rho_h \mid \tilde{m}_{i,j} = h) = \frac{\alpha_{i,j}\pi_{i,j}}{\alpha_{i,j}\pi_{i,j} + (1-\alpha_{i,j})(1-\pi_{i,j})}, \\ \pi_{i,j}^l &= Pr(\tilde{\rho}_{i,j} = \rho_h \mid \tilde{m}_{i,j} = l) = \frac{(1-\alpha_{i,j})\pi_{i,j}}{(1-\alpha_{i,j})\pi_{i,j} + \alpha_{i,j}(1-\pi_{i,j})}. \end{aligned} \tag{27}$$

These individual probabilities can then be aggregated to calculate $P(s|m)$, which is the probability that the economy is in state $s$ conditional on receiving a vector $m$ of messages about all bilateral relationships in the economy. These updated probabilities allow for the derivation of the updated price function, similar to equation (26), for a given disclosure policy and vector of messages:

$$P_i(\alpha, m) = \frac{1}{R} \sum_{s=1}^{S} \frac{P(s|m)e^{\phi(s)}}{\sum_{k=1}^{S} P(k|m)e^{\phi(k)}} \left( \bar{V} + \bar{\delta} \sum_{j \neq i} \rho_{i,j}(s) - \frac{\gamma}{N} \left( \sigma_V^2 + 2\sigma_\delta^2 \sum_{j \neq i} \rho_{i,j}(s)^2 \right) \right), \tag{28}$$

where $\alpha$ is a vector containing all bilateral disclosure decisions $\alpha_{i,j}$. We are not able to solve the disclosure decision analytically for the N-firm setting. However, we are able to derive a condition for a symmetric equilibrium with a corner disclosure policy, where all bilateral disclosure decisions are identical such that $\alpha_{i,j} = \alpha^*$, and have a corner disclosure policy, where $\alpha^*$ is either $\alpha^* = 0.5$ or $\alpha^* = 1$. The corner disclosure policy is the optimal policy in the benchmark model. In a symmetric equilibrium with a corner disclosure policy, firms will choose a non-disclosure policy $\alpha^* = 0.5$ if and only if $E_m[P_i(\alpha^* = 0.5, m)] - E_m[P_i(\alpha^* = 1, m)] > 0$, which is satisfied when

$$\sum_{s=1}^{S} \left( \frac{P(s)e^{\phi(s)}}{\sum_{k=1}^{S} P(k)e^{\phi(k)}} - P(s) \right) \left( \bar{V} + \bar{\delta} \sum_{j \neq i} \rho_{i,j}(s) - \frac{\gamma}{N} \left( \sigma_V^2 + 2\sigma_\delta^2 \sum_{j \neq i} \rho_{i,j}(s)^2 \right) \right) > 0.$$

#### 6.1.1 Numerical results

While we provide analytical conditions for the general disclosure decision above, it is quite abstract so we resort to illustrating by numerical examples in this part. Due to the large number of parameters, it is challenging even for numerical illustration. To simplify the process of solving for optimal disclosure, motivated by the analysis above, we will only consider symmetric equilibria with a corner disclosure policy such that $\alpha^* = \alpha_{i,j}$ for a given $N$. We will compare if $\alpha_{i,j} = 1$ or $\alpha_{i,j} = 0.5$ maximizes the sum of expected prices conditional on $\alpha^* = 0.5$ or $\alpha^* = 1$ respectively. Solutions for such equilibria are examined through the following parameters: $\gamma = 1$, $\pi_{i,j} = \frac{1}{2}$ $\forall i$ and $j$, $\rho_h = 1$, $\rho_l = 0.1$, $\overline{V} = 1$, $\sigma_V = 0.1$, $R = 1$, $\overline{\delta} = \frac{D}{N}$.

To guide the numerical analysis, we will test the following. Proposition 2 posits that optimal disclosure is equal to 0.5 iff $\frac{\rho_h + \rho_l}{2} < \frac{\bar{\delta}}{\gamma \sigma_\delta^2} < \rho_h + \rho_l$ for the two asset model. We conjecture that this relation also dictates optimal disclosure for arbitrary $N$ assets.

To test the conjecture, we input parameter values into Proposition 2 to find:

$$0.55 < \frac{D}{N\sigma_\delta^2} < 1.1. \tag{29}$$

We can now leverage Proposition 2 from the main setting to examine optimal disclosure for various levels of $D$, $\sigma_\delta^2$, and most importantly $N$. To do this, we use the following steps. First, we compute a list of $\sigma_\delta^2$ values: $[1, 3, 5, 7, 9]$. Next, we compute the upper and lower bounds for $D$ using proposition 2. We then run the numerical exercise for $D$ equal to 1% higher or lower than the Proposition 2 bounds. If the resulting equilibrium is $\alpha_{i,j} = \alpha^* = 0.5$ then it is marked in blue, if the resulting equilibrium is $\alpha_{i,j} = \alpha^* = 1$ then it is marked with red.

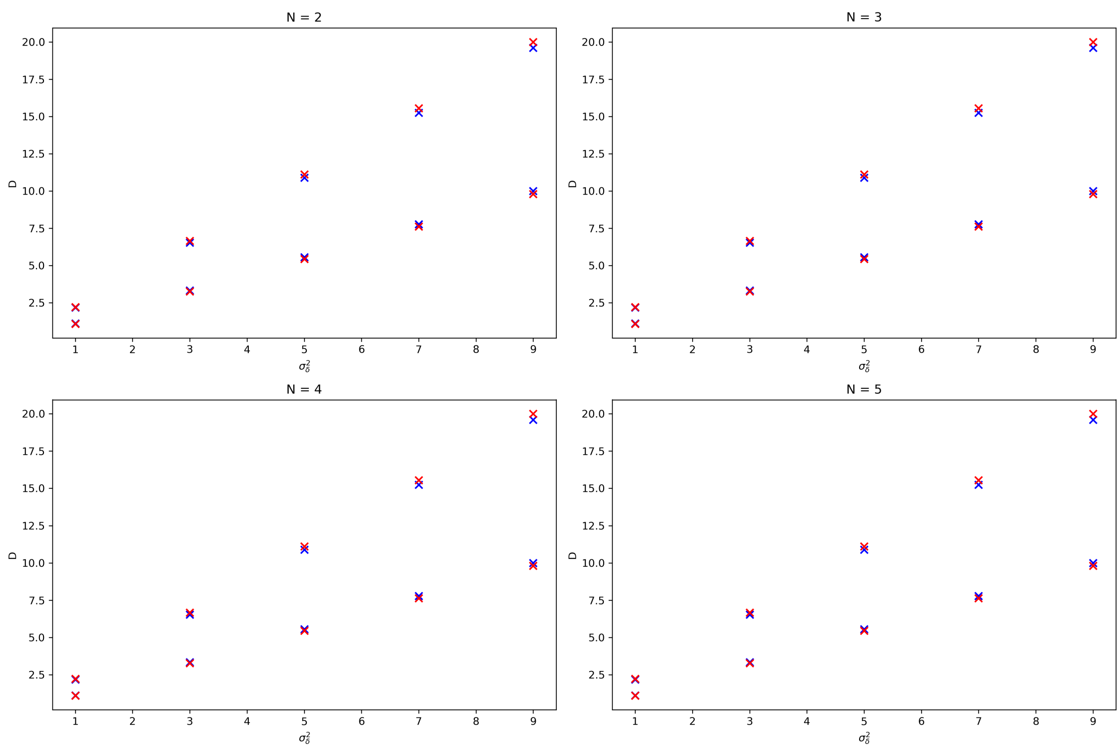



Figure 9: $D$ Set within and outside Proposition 2 Boundaries

As we can see, parametrizations set within the boundaries of Proposition 2 have optimal $\alpha_{i,j} = \alpha^* = 0.5$ equilibria whereas the parametrizations set outside the boundaries have $\alpha_{i,j} = \alpha^* = 1$ equilibria. Therefore, even though the conditions for the N-asset case are quite complicated, with some simplifying assumptions, our numerical analysis shows that Proposition 2 can be generalized to the large economy version of the model.

## 7 Introducing Correlation between Firm-Specific Cash Flows and Cash Flows from Relationship Formation

So far in our analysis we assume that the firm-specific cash flow component before forming a particular relationship, $\tilde{V}_i$, and the cash flow component coming from that particular relationship formation, $\tilde{\Delta}$, are mutually independent. In reality, it is quite plausible that those cash flow components can be correlated. For example, suppose a cloud service provider has several big retailers as customers. Then, securing another retailer as an additional customer may result in the existing cash flow from the cloud service provider being even more positively correlated with the cash flow from the additional retailer customer and thus the diversification cost becoming even bigger, whereas securing a luxury goods retailer may result in the cash flows being negatively correlated, to the extent that luxury goods consumption is relatively recession-proof.

In this section we build on the original setting from Section 3 and introduce correlation between $\tilde{V}_i$ and a component of $\tilde{\Delta}$, $\tilde{\delta}$. Denote Cov$(\tilde{V}_i, \tilde{\delta}) = \sigma_{\delta i}$.[9] To keep the analysis simple yet still informative, we assume that $\sigma_{\delta A} = \sigma_{\delta B} \equiv \sigma_{\delta \iota}$. The price $P_j$ for any firm $j \in \{A, B\}$ is given by

$$P_i = \frac{\bar{V} - \frac{\gamma}{2}\sigma_V^2}{R} + \frac{\bar{\delta} - \gamma\sigma_{\delta\iota}}{R}\left[\frac{\pi\rho_h e^{-\gamma\bar{\delta}\rho_h + \frac{\gamma^2}{2}(\rho_h^2\sigma_\delta^2 + \rho_h\sigma_{\delta\iota})} + (1-\pi)\rho_l e^{-\gamma\bar{\delta}\rho_l + \frac{\gamma^2}{2}(\rho_l^2\sigma_\delta^2 + \rho_l\sigma_{\delta\iota})}}{\pi e^{-\gamma\bar{\delta}\rho_h + \frac{\gamma^2}{2}(\rho_h^2\sigma_\delta^2 + \rho_h\sigma_{\delta\iota})} + (1-\pi) e^{-\gamma\bar{\delta}\rho_l + \frac{\gamma^2}{2}(\rho_l^2\sigma_\delta^2 + \rho_l\sigma_{\delta\iota})}}\right]$$
$$- \frac{\gamma\sigma_\delta^2}{R}\left[\frac{\pi\rho_h^2 e^{-\gamma\bar{\delta}\rho_h + \frac{\gamma^2}{2}(\rho_h^2\sigma_\delta^2 + \rho_h\sigma_{\delta\iota})} + (1-\pi)\rho_l^2 e^{-\gamma\bar{\delta}\rho_l + \frac{\gamma^2}{2}(\rho_l^2\sigma_\delta^2 + \rho_l\sigma_{\delta\iota})}}{\pi e^{-\gamma\bar{\delta}\rho_h + \frac{\gamma^2}{2}(\rho_h^2\sigma_\delta^2 + \rho_h\sigma_{\delta\iota})} + (1-\pi) e^{-\gamma\bar{\delta}\rho_l + \frac{\gamma^2}{2}(\rho_l^2\sigma_\delta^2 + \rho_l\sigma_{\delta\iota})}}\right].$$

The results on relationship disclosure are summarized in the following Proposition:

[9] We still asume that $\widetilde{V}_A$ and $\widetilde{V}_B$ are still independent, implying that the correlation between $\widetilde{V}_i$ and $\widetilde{\delta}$ cannot be too large.

**Proposition 6.** *The optimal disclosure policy is*

$$\alpha^* = \begin{cases} \frac{1}{2}, \text{ if } \frac{1}{2}[(\rho_h + \rho_l) - \frac{\sigma_{\delta\iota}}{\sigma_\delta}] < \frac{(\bar{\delta} - \gamma\sigma_\delta)}{\gamma\sigma_\delta^2} < (\rho_h + \rho_l), \\ 1, \text{ otherwise.} \end{cases}$$

Proposition 6 shows that the results are largely in line with that in the main setting: the relationship increases the expected cash flow but also increases the diversification cost due to the common component of the cash flow. The introduction of correlation between $\tilde{V}_i$ and $\tilde{\delta}$, however, also generates some subtle differences in our results.

If the correlation is positive, $\sigma_{\delta\iota} > 0$, then intuitively, the driving forces with positive correlation are similar to that in our main setting: relationship disclosure increases the expected cash flow due to the matching profitability but also exacerbates the diversification cost due to the collaboration risk, even more so due to the positive correlation. Therefore, the range of no disclosure actually increases.

If the correlation is negative, then $\sigma_{\delta\iota} < 0$. In this case relationship disclosure actually reduces the diversification cost due to a reduction of the collaboration risk, resulting in the firm more likely to disclose. In the extreme case, where the negative correlation is so large as to dominate the increase in diversification cost due to the common component $\tilde{\Delta}$ (which is determined by $\rho_h$ and $\rho_l$), then relationship disclosure only has benefits and the firm will therefore always disclose.

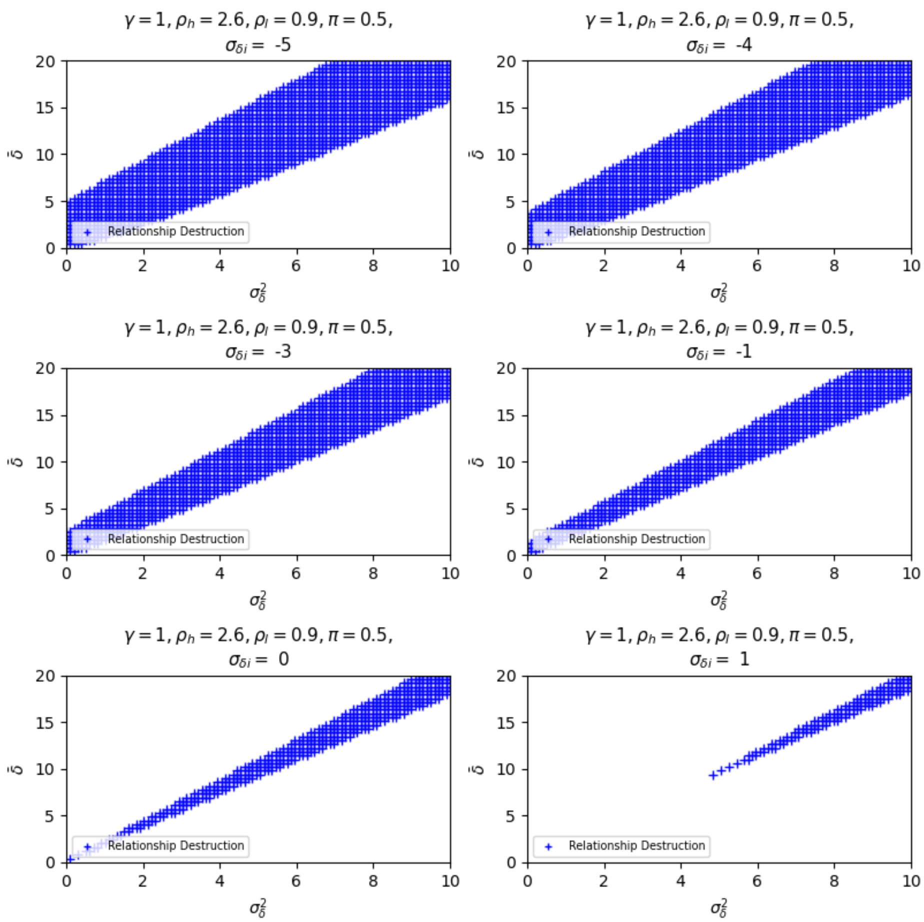
If we allow for relationship formation as in Section 5, then mandatory disclosure destroys relationships if and only if

$$\frac{(\pi\rho_h^2 + (1-\pi)\rho_l^2)}{(\pi\rho_h + (1-\pi)\rho_l)} > \frac{(\bar{\delta} - \gamma\sigma_\delta)}{\gamma\sigma_\delta^2}$$
$$> max\left\{ \frac{1}{2}[(\rho_h + \rho_l) - \frac{\sigma_{\delta\iota}}{\sigma_\delta}], \frac{\pi\rho_h^2 e^{-\gamma\bar{\delta}\rho_h + \frac{\gamma^2}{2}(\rho_h^2\sigma_\delta^2 + \rho_h\sigma_{\delta\iota})} + (1-\pi)\rho_l^2 e^{-\gamma\bar{\delta}\rho_l + \frac{\gamma^2}{2}(\rho_l^2\sigma_\delta^2 + \rho_l\sigma_{\delta\iota})}}{\pi\rho_h e^{-\gamma\bar{\delta}\rho_h + \frac{\gamma^2}{2}(\rho_h^2\sigma_\delta^2 + \rho_h\sigma_{\delta\iota})} + (1-\pi)\rho_l e^{-\gamma\bar{\delta}\rho_l + \frac{\gamma^2}{2}\left(\rho_l^2\sigma_\delta^2 + \rho_l\sigma_{\delta\iota}\right)}} \right\}.$$

Again, the results are largely in line with that in the main setting. Positive correlation between $\tilde{V}_i$ and $\tilde{\delta}$ results in the firm less likely to form a relationship as diversification cost from the common relationship increases even more, whereas negative correlation results in the firm more likely to form a relationship, and there will be circumstances when mandatory relationship destroys disclosure formation in the first place, so long as $\sigma_{\delta\iota}$ is not too negative.

Due to the complexities of the expressions, we resort to numerical examples to illustrate our results. Figure 10 shows the equilibrium relationship formation and when mandatory disclosure destroys relationship formation. For a broad range of parameter values, we can see that mandatory disclosure destroys relationship formation when the matching profitability of relationship formation relative to the collaboration risk (after properly accounting for the correlation between $\tilde{V}_i$ and $\tilde{\delta}$) is in the intermediate region, in line with our previous findings.

Figure 10: Mandatory Disclosure and Destruction of Relationships in the Correlation Model



# 8 Welfare Analysis of Mandatory Relationship Intensity Disclosure

So far we have been focusing on how relationship disclosure affects the cost of capital or asset prices. However, in a setting with real decisions that affect cash flows, asset prices are not equivalent to measures of investors' welfare (e.g., Gao, 2010). In addition, in the main setting we focus on the cost of relationship disclosure in affecting firm prices, without considering the potential benefit of relationship disclosure in reducing the uncertainty faced by risk-averse investors and a potential increase in investors' welfare. In this section we study whether mandatory disclosure regulation increases or decreases investor welfare, i.e., the expected utility. When firms choose to voluntarily disclose a relationship, such regulation clearly does not affect welfare, so the interesting question is how such regulation affects welfare when firms voluntarily choose not to disclose relationship.

Following Diamond (1985), we endow the representative investor with the initial shares to have a complete welfare analysis. This assumption implies that $W_0 = 1/2P_A + 1/2P_B$. We can then write final wealth in equation (2) as

$$\tilde{W} = R\left(\frac{1}{2}P_A + \frac{1}{2}P_B\right) + q_A(\tilde{F}_A - RP_A) + q_B(\tilde{F}_B - RP_B).$$

If we impose the market clearing condition $q_A = q_B = 1/2$, which must be satisfied in equilibrium, we have that final wealth in equilibrium is given by $\tilde{W} = \frac{1}{2}(\tilde{F}_A + \tilde{F}_B)$.

The following proposition shows that mandatory disclosure regulation may decrease investor welfare when such disclosure results in destruction of relationship formation, but does not change investor welfare when such disclosure does not result in destruction of relationship formation.

**Proposition 7.** *Mandatory disclosure does not affect investor welfare when firms are forced to dis-*

*close relationship intensities without destroying the underlying relationship formation. However, if*

$$\max\left\{\frac{\rho_h+\rho_l}{2}, \frac{\pi\rho_h^2 e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2-\gamma\rho_h\bar{\delta}}+(1-\pi)\rho_l^2 e^{\frac{1}{2}\gamma^2\rho_l^2\sigma_\delta^2-\gamma\rho_l\bar{\delta}}}{\pi\rho_h e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2-\gamma\rho_h\bar{\delta}}+(1-\pi)\rho_l e^{\frac{1}{2}\gamma^2\rho_l^2\sigma_\delta^2-\gamma\rho_l\bar{\delta}}}\right\} < \frac{\bar{\delta}}{\gamma\sigma_\delta^2} < \frac{\pi\rho_h^2+(1-\pi)\rho_l^2}{\pi\rho_h+(1-\pi)\rho_l},$$

*then the investor is strictly worse off when relationship formation is destroyed.*

Intuitively, Proposition 7 states that mandatory relationship intensity disclosure does not have any welfare benefit. In the best case scenario it keeps the same welfare as before the regulation when relationships are not destroyed. However, when relationships are destroyed due to the regulation, then mandatory disclosure makes investors worse off. The conditions in Proposition 7 regarding when mandatory disclosure destroys welfare are therefore exactly the same as that in Proposition 4. The shaded regions shown in figure 8 where mandatory disclosure destroys relationship formation are also the regions when mandatory disclosure reduces welfare.

# 9 Empirical and Regulatory Implications

Our results provide several implications related to disclosure of relationships among firms. First, Corollaries 1 and 2 and the numerical examples offer some predictions on price responses to relationship disclosure. We show that price response to relationship intensity disclosure will be lower if the relationship is riskier, and will be higher if the expected levels of matching intensity and matching profitability are higher. To the extent that expected levels of matching intensity and matching profitability are more important for small firms' cash flows, the price response will be higher in magnitude for smaller firms relative to larger firms in the relationship. Interestingly, Corollary 2 also suggests that disclosure of a more intensive relationship (than expected) may not necessarily generate a positive price response, if the relationship turns out to be very risky relative to the expected benefits.

Second, we show that relationship disclosure increases the ex-ante perceived risk of firms, due to a decrease in diversification benefit, which may result in lower ex-ante prices. Therefore, we predict that firms making relationship disclosure are more risky than firms that do not make such disclosure, which can be empirically proxied by, e.g., volatility of returns, operating earnings, or cash flows from operations.

Third, we show that firms that choose to voluntarily disclose relationship have either high or low expected benefits from such relationship. To the extent that the benefits from relationship translate into higher future earnings and cash flows, our results imply that in the pre-SFAS 131 regime when relationship disclosures are voluntary, firms that choose to disclose relationship would have either high or low future expected earnings and cash flows from operations whereas firms that choose not to disclose such relationship will have intermediate future expected earnings and cash flows.

Fourth, our analysis of N-firm settings shows that even in a large economy, relationship risk is priced. This suggests that, data permitting, one can calculate a relationship-based risk factor based on the firms' disclosed relationship with major customers and suppliers. The higher exposure an firm is to such risk factor, the higher expected return the firm is, controlling for other risk factors.

Fifth, our analysis of the setting with correlation between $\tilde{V}_i$ and $\tilde{\delta}$ in Section 7 suggests that the firm is more likely to make relationship intensity disclosure if the correlation of the cash flow generated between the new relationship and existing relationships is more negative (or less likely to be positive). Therefore, conditional on firm disclosing a new customer relationship, such relationship should generate higher expected future cash flows if the customer is more correlated with the firms' existing customers, which can be proxied by, e.g., correlation of stock returns, correlation of past earnings.

Finally, we show that mandating relationship intensity disclosure has the unintended consequences of discouraging firms to form beneficial relationships in the first place, which should be of interests to regulators contemplating more such disclosures. The results also imply that firms who voluntarily disclose in the pre-SFAS 131 regime will make less relationship disclosure post-SFAS 131, when such disclosure becomes mandatory. However, those firms will also have higher profitability post SFAS-131 as relationships that only have high benefits will be formed. In addition, results from Section 8 suggest that such mandatory disclosure will result in a lower investor welfare if and only if it destroys relationship formation, which may have macroeconomic implications regarding, e.g., consumption growth and GDP. When such disclosure does not result in destroying relationship formation, mandatory disclosure cannot decrease investor welfare.

## 10 Conclusion

In this paper, we examine how the payoff correlation structure is endogenously generated and study the incentives and implications of firms to form and disclose such correlations. Since asset payoffs are cash flows generated by production, in principle, any payoff correlation structure must trace back to the production process. In this paper, we open the black box of the production process from a particular perspective, namely, firm relationships.

This paper develops a new conceptual framework to analyze the incentives of firms to form and disclose relationships and its implications for asset prices, abstracting away from the proprietary cost framework on which most prior literature focuses. Forming a relationship generates synergies between firms. But relationship formations also incur a cost. Relationship formation makes the performance of the firms correlated, reducing the ability to hedge the risk of investing in the firms and generating additional risk in financial markets. We first study the trade-offs on asset prices when there is uncertainty about the relationship between two firms. Having a relationship generates two effects on the cash flows of the firms. First, the cash flows of the two firms have an additional payoff component with a positive mean. Second, the cash flows also become more correlated. On the one hand, the increase in the mean of asset payoffs raises the investor's perceived returns on investing in risky assets and thus increases her demand for the assets and their prices. On the other hand, the increase in the variance of the asset payoffs decreases the investor's ability to diversify her portfolio as cash flows are now correlated, which decreases the investor's demand for the assets and their prices.

We further analyze the optimal voluntary disclosure policies about a relationship. Unlike previous literature, disclosing a relationship in our framework is about disclosing the value of a common component in the asset payoffs of the two firms, generating a correlation structure between the two firms. It is not about disclosing the realization of the fundamentals as in most research on disclosure. We finally examine under which conditions firms choose to form relationships and their matching profitability. Our analysis provides insights for regulations mandating relationship disclosure, by highlighting the consequences of such regulations for asset prices and relationship formation, as well as proposing an endogenous and non-proprietary cost of such disclosure.

As a final remark, in this paper we consider a two-firm setting for the sake of tractability. This setting can be literally interpreted as two big representative firms. For example, Walmart and Pepsi can be considered as two big representative firms in their respective industries. In a large economy with N firms, the number of relationships among firms is large and modelling relationship between any two of the N firms becomes quite challenging. While we make some attempts in Section 6 to show that our results largely hold with appropriate extensions of relationship formation to N-firm settings and that relationship risk is priced, there is always the issue that whether the extension to the N-firm setting correctly captures the interactions of N firms in reality. We leave that for future work but conjecture that our results will hold so long as the modelling of relationship among firms results in relationship being priced in equilibrium, which is supported by the empirical evidence in Herskovic (2018).

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# Appendix

## Statistical Properties of Cash Flows

The common component of the cash flows $\tilde{\Delta}$ follows a mixture of two Normal distributions. Hence, $\tilde{\Delta}$ does not follow a Normal distribution. Since $\tilde{\rho}$ affects both the mean and the variance of $\tilde{\Delta}$, uncertainty about $\tilde{\rho}$ generates uncertainty about both the mean and the variance of the common component in the cash flows. Hence, states of the world with high mean will also have high variance, generating skewness in the distribution of the common component.

The sign of skewness of the common component of the cash flows is going to be given by the sign of the following expression:

$$E\left[(\tilde{\Delta}-E[\tilde{\Delta}])^3\right]=\pi(1-\pi)(\rho_h-\rho_l)\bar{\delta}[(1-2\pi)(\rho_h-\rho_l)^2\bar{\delta}+3(\rho_h^2-\rho_l^2)\sigma_\delta^2].$$

Skewness is a measure of the asymmetry of the distribution of cash flows. For $\pi=0.5$, the skewness will always be strictly positive. Risk averse investors have preferences over higher moments of the distribution. Specifically, risk averse investors value more assets with positive skewness, and it will be reflected positively in asset prices. There will only be negative skewness when both $\pi>0.5$ and $\bar{\delta}$ is large. The uncertainty about the variance of $\tilde{\Delta}$ will also generate kurtosis in the distribution of the common component. Kurtosis increases the probability of extreme outcomes and will lead to a decrease in asset prices as a result.

In addition, the distribution of the common component $\tilde{\Delta}$ may be unimodal or bimodal depending on the parameters of the distribution. Define $d\equiv\frac{(\rho_h-\rho_l)\bar{\delta}}{2\sigma_\delta^2\sqrt{\rho_h\rho_l}}$. The distribution of the common component $\tilde{\Delta}$ is unimodal if and only if $d\leq 1$ or

$$|\ln(1-\pi)-\ln(\pi)|\geq 2\log\left(d-\sqrt{d^2-1}\right)+2d\sqrt{d^2-1}.$$

For sufficiently low $\bar{\delta}$, the distribution of the common component $\tilde{\Delta}$ will be unimodal and the distribution will have positive skewness as described above. For sufficiently high $\bar{\delta}$ and if $\pi>0.5$, the distribution of the common component will be bimodal and have negative skewness. For $\pi\leq 0.5$ and large $\bar{\delta}$, the distribution of the common component will be bimodal and have positive skewness.

## Proof of Proposition 1

The representative investor maximizes equation (5) with respect to $q_A$ and $q_B$. The first-order condition (FOC) with respect to $q_A$ is given by

$$\pi \exp\left[-\gamma\left(\mu_W(h) - \frac{\gamma\sigma_W^2(h)}{2}\right)\right](\bar{V} + \rho_h\bar{\delta} - RP_A - \gamma q_A(\sigma_V^2 + \rho_h^2\sigma_\delta^2) - \gamma q_B\rho_h^2\sigma_\delta^2 +$$
$$(1-\pi)\exp\left[-\gamma\left(\mu_W(l) - \frac{\gamma\sigma_W^2(l)}{2}\right)\right](\bar{V} + \rho_l\bar{\delta} - RP_A - \gamma q_A(\sigma_V^2 + \rho_l^2\sigma_\delta^2) - \gamma q_B\rho_l^2\sigma_\delta^2 = 0.$$

We can get the FOC with respect to $q_B$ in the same way. When we plug the market clearing conditions $q_A = \frac{1}{2}$ and $q_B = \frac{1}{2}$ into both conditions, we get prices for asset $j \in \{A, B\}$:

$$\begin{aligned} P_j &= \frac{\bar{V} - \frac{1}{2}\gamma\sigma_V^2}{R} + \\ &+ \frac{\bar{\delta}}{R}\left[\frac{\pi\rho_h e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2 - \gamma\rho_h\bar{\delta}} + (1-\pi)\rho_l e^{\frac{1}{2}\gamma^2\rho_l^2\sigma_\delta^2 - \gamma\rho_l\bar{\delta}}}{\pi e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2 - \gamma\rho_h\bar{\delta}} + (1-\pi) e^{\frac{1}{2}\gamma^2\rho_l^2\sigma_\delta^2 - \gamma\rho_l\bar{\delta}}}\right] \\ &- \frac{\gamma\sigma_\delta^2}{R}\left[\frac{\pi\rho_h^2 e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2 - \gamma\rho_h\bar{\delta}} + (1-\pi)\rho_l^2 e^{\frac{1}{2}\gamma^2\rho_l^2\sigma_\delta^2 - \gamma\rho_l\bar{\delta}}}{\pi e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2 - \gamma\rho_h\bar{\delta}} + (1-\pi) e^{\frac{1}{2}\gamma^2\rho_l^2\sigma_\delta^2 - \gamma\rho_l\bar{\delta}}}\right]. \end{aligned}$$

## Proof of Corollary 2

We know that $\pi^h \geq \pi^l$ as $\alpha \geq \frac{1}{2}$. In addition, from equation (14) $\pi^{ram}$ is increasing in $\pi^m$ as its inverse is decreasing in $\pi^m$. Therefore, from equation (15) $\tilde{m} = h$ increases prices (and $\tilde{m} = l$ decreases prices) if and only if $P_j(\alpha; \tilde{m})$ is increasing in $\pi^{ram}$. From equation (13), the derivative of $P_j(\alpha; \tilde{m})$ with respect to $\pi^{ram}$ is

$$\begin{aligned} & \frac{\bar{\delta}}{R}(\rho_h - \rho_l) - \frac{\gamma\sigma_\delta^2}{R}(\rho_h^2 - \rho_l^2) \\ = & \frac{\rho_h - \rho_l}{R\gamma\sigma_\delta^2}[\frac{\bar{\delta}}{\gamma\sigma_\delta^2} - (\rho_h + \rho_l)] \\ > & \ 0, \end{aligned}$$

if and only if $\frac{\bar{\delta}}{\gamma\sigma_\delta^2} > \rho_h + \rho_l$.

## Proof of Proposition 2

The FOC of the expected asset price (15) with respect to $\alpha$ equals zero at $\alpha = \frac{1}{2}$ is:

$$\left.\frac{\partial E[P_j(\alpha; \tilde{m})]}{\partial \alpha}\right|_{\alpha=\frac{1}{2}} = 0.$$

The relevant expression for the sign of the second-order condition (SOC) $\frac{\partial^2 E[P_j(\alpha;\tilde{m})]}{\partial \alpha^2}$ for $\alpha = \frac{1}{2}$ to be a maximum is given by

$$\left[\gamma\sigma_\delta^2(\rho_h + \rho_l) - \bar{\delta}\right] \left(e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2 - \gamma\rho_h\bar{\delta}} - e^{\frac{1}{2}\gamma^2\rho_l^2\sigma_\delta^2 - \gamma\rho_l\bar{\delta}}\right) < 0;$$

otherwise $\alpha = \frac{1}{2}$ is a minimum. There are two ways under which the inequality above is satisfied for $\alpha = \frac{1}{2}$ to be the maximum:

1. If both of these conditions are satisfied: $\bar{\delta} > \gamma\sigma_\delta^2(\rho_h + \rho_l)$ and $e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2 - \gamma\rho_h\bar{\delta}} > e^{\frac{1}{2}\gamma^2\rho_l^2\sigma_\delta^2 - \gamma\rho_l\bar{\delta}}$. These two conditions are satisfied if and only if $\frac{1}{2}\gamma(\rho_l + \rho_h)\sigma_\delta^2 > \bar{\delta} > \gamma(\rho_l + \rho_h)\sigma_\delta^2$. This is not a feasible condition.

2. If both of these conditions are satisfied: $\bar{\delta} < \gamma\sigma_\delta^2(\rho_h + \rho_l)$ and $e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2 - \gamma\rho_h\bar{\delta}} < e^{\frac{1}{2}\gamma^2\rho_l^2\sigma_\delta^2 - \gamma\rho_l\bar{\delta}}$. These two conditions are satisfied if and only if $\frac{1}{2}\gamma(\rho_l + \rho_h)\sigma_\delta^2 < \bar{\delta} < \gamma(\rho_l + \rho_h)\sigma_\delta^2$. This is a feasible condition.

Hence, we can conclude that $\alpha = \frac{1}{2}$ is the maximum if and only if $\frac{1}{2}\gamma(\rho_l + \rho_h)\sigma_\delta^2 < \bar{\delta} < \gamma(\rho_l + \rho_h)\sigma_\delta^2$. Otherwise, $\alpha = 1$ is the maximum since $\alpha = \frac{1}{2}$ is the minimum and $\frac{\partial E[P_j(\alpha;\tilde{m})]}{\partial \alpha} > 0$ for any $\alpha \in (\frac{1}{2}, 1]$.

## Proof of Corollary 3

When the optimal policy is non-disclosure ($\alpha^* = \frac{1}{2}$), the message received by the representative investor contains no information and we are in the scenario of Section 3.3, where the asset price is given by equation (6). In this case, the price is independent of the message, and thus $E[P_j(\alpha^* = \frac{1}{2}; \tilde{m})]$ is also given by equation (6). Instead, when the optimal policy is full disclosure ($\alpha^* = 1$), the message received by the representative investor contains full information and asset prices are given by equation (4) depending on the realization of $\tilde{\rho}$. Using equation (15), we can calculate the expected asset price as

$$E[P_j(\alpha^* = 1; \tilde{m})] = \pi P_j^{PI}(\tilde{\rho} = \rho_h) + (1 - \pi)P_j^{PI}(\tilde{\rho} = \rho_l),$$

where we have used that $Pr(\tilde{m} = h) = \pi$, $Pr(\tilde{m} = l) = 1 - \pi$, $P_j(\alpha^* = 1; \tilde{m} = h) = P_j^{PI}(\tilde{\rho} = \rho_h)$, and $P_j(\alpha^* = 1; \tilde{m} = l) = P_j^{PI}(\tilde{\rho} = \rho_l)$.

## Proof of Lemma 1

From Figure 6, we can see that no firm has any incentive to deviate from a no-relationship equilibrium as there is no benefit from an individual deviation.

## Proof of Proposition 3

There exists a Pareto-dominant relationship equilibrium in which firms will form a relationship with full-disclosure ($\alpha^* = 1$) if and only if the following two conditions are satisfied: 1) using equations (20) and (22), for any firm $j \in \{A, B\}$, we have that $P_j^{NR} < E[P_j(\alpha^* = 1; \tilde{m})]$; and 2) $\alpha^* = 1$ is optimal following Proposition 2. These two conditions are satisfied if and only if

$$\rho_h + \rho_l < \frac{\bar{\delta}}{\gamma \sigma_\delta^2},$$

or

$$\frac{\pi \rho_h^2 + (1-\pi)\rho_l^2}{\pi \rho_h + (1-\pi)\rho_l} < \frac{\bar{\delta}}{\gamma \sigma_\delta^2} < \frac{\rho_h + \rho_l}{2}.$$

There exists a Pareto-dominant relationship equilibrium in which firms will form a relationship with non-disclosure ($\alpha^* = \frac{1}{2}$) if and only if the following two conditions are satisfied: 1) using equations (6) and (22), for any firm $j \in \{A, B\}$, we have that $P_j^{NR} < E[P_j(\alpha^* = \frac{1}{2}; \tilde{m})]$; and 2) $\alpha^* = \frac{1}{2}$ is optimal following Proposition 2. These two conditions are satisfied if and only if

$$\max\left\{\frac{\rho_h + \rho_l}{2}, \frac{\pi \rho_h^2 e^{\frac{1}{2}\gamma^2 \rho_h^2 \sigma_\delta^2 - \gamma \rho_h \bar{\delta}} + (1-\pi)\rho_l^2 e^{\frac{1}{2}\gamma^2 \rho_l^2 \sigma_\delta^2 - \gamma \rho_l \bar{\delta}}}{\pi \rho_h e^{\frac{1}{2}\gamma^2 \rho_h^2 \sigma_\delta^2 - \gamma \rho_h \bar{\delta}} + (1-\pi)\rho_l e^{\frac{1}{2}\gamma^2 \rho_l^2 \sigma_\delta^2 - \gamma \rho_l \bar{\delta}}}\right\} < \frac{\bar{\delta}}{\gamma \sigma_\delta^2} < \rho_h + \rho_l.$$

## Proof of Proposition 4

Mandatory disclosure will destroy relationships if and only if the following two conditions are satisfied: 1) for any firm $j \in \{A, B\}$ we have that $E[P_j(\alpha = \frac{1}{2}; \tilde{m})] > P_j^{NR} > E[P_j(\alpha = 1; \tilde{m})]$; and 2) it is optimal for firms not to disclose, hence $\alpha^* = \frac{1}{2}$ is optimal following Proposition 2. These two conditions are satisfied if and only if

$$\max\left(\frac{\rho_h + \rho_l}{2}, \frac{\pi \rho_h^2 e^{\frac{1}{2}\gamma^2 \rho_h^2 \sigma_\delta^2 - \gamma \rho_h \bar{\delta}} + (1-\pi)\rho_l^2 e^{\frac{1}{2}\gamma^2 \rho_l^2 \sigma_\delta^2 - \gamma \rho_l \bar{\delta}}}{\pi \rho_h e^{\frac{1}{2}\gamma^2 \rho_h^2 \sigma_\delta^2 - \gamma \rho_h \bar{\delta}} + (1-\pi)\rho_l e^{\frac{1}{2}\gamma^2 \rho_l^2 \sigma_\delta^2 - \gamma \rho_l \bar{\delta}}}\right) < \frac{\bar{\delta}}{\gamma \sigma_\delta^2} < \frac{\pi \rho_h^2 + (1-\pi)\rho_l^2}{\pi \rho_h + (1-\pi)\rho_l}.$$

For the inequalities to be satisfied, we need

$$\frac{\pi\rho_h^2 e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2-\gamma\rho_h\bar{\delta}}+(1-\pi)\rho_l^2 e^{\frac{1}{2}\gamma^2\rho_l^2\sigma_\delta^2-\gamma\rho_l\bar{\delta}}}{\pi\rho_h e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2-\gamma\rho_h\bar{\delta}}+(1-\pi)\rho_l e^{\frac{1}{2}\gamma^2\rho_l^2\sigma_\delta^2-\gamma\rho_l\bar{\delta}}}<\frac{\pi\rho_h^2+(1-\pi)\rho_l^2}{\pi\rho_h+(1-\pi)\rho_l},$$

which is satisfied when

$$\frac{\bar{\delta}}{\gamma\sigma_\delta^2}>\frac{\rho_h+\rho_l}{2}.$$

For the inequalities to be satisfied, we also need

$$\frac{\rho_h+\rho_l}{2}<\frac{\pi\rho_h^2+(1-\pi)\rho_l^2}{\pi\rho_h+(1-\pi)\rho_l},$$

which is satisfied when

$$\pi\rho_h>(1-\pi)\rho_l.$$

## Proof of Proposition 5

Taking the FOC of equation (24) with respect to $q_i$ and setting $q_i=\frac{1}{N}$ for all $i$ , we obtain:

$$\gamma e^{-\gamma[W_0R+\bar{V}-\frac{\gamma\sigma_V^2}{2N}-\frac{R}{N}\sum_{i=1}^N P_i]}\sum_{s=1}^S P(s)e^{\phi(s)}\left(\bar{V}+\bar{\delta}\sum_{j\neq i}\rho_{i,j}(s)-RP_i-\frac{\gamma}{N}\left(\sigma_V^2+2\sigma_\delta^2\sum_{j\neq i}\rho_{i,j}(s)^2\right)\right)=0.$$

If we isolate the price $P_i$, we get equation (26).

## Proof of Corollary 4

If we take the limit when $N\to\infty$ of the price equation (26), we obtain:

$$\lim_{N\to\infty}P_i=\frac{\bar{V}}{R}+\frac{D\mathbb{E}_{\mathbb{P}}(\bar{\rho}_i)}{R}-\frac{2\gamma\sigma_\delta^2\mathbb{E}_{\mathbb{P}}(\bar{\rho_i^2})}{R}<\infty,$$

where

$$\mathbb{E}_{\mathbb{P}}(\bar{\rho}_i)=\frac{1}{N-1}\sum_{s=1}^S\frac{P(s)e^{\phi(s)}}{\sum_{k=1}^S P(k)e^{\phi(k)}}\sum_{j\neq i}\rho_{i,j}(s),$$

and

$$\mathbb{E}_{\mathbb{P}}(\bar{\rho}_i)^2=\frac{1}{N-1}\sum_{s=1}^S\frac{P(s)e^{\phi(s)}}{\sum_{k=1}^S P(k)e^{\phi(k)}}\sum_{j\neq i}\rho_{i,j}(s)^2.$$

## Proof of Proposition 6

Denote Cov($\tilde{V}_i, \tilde{\delta}$) = $\sigma_{\delta i}$. We still assume that $\tilde{V}_1$and $\tilde{\delta}$ are jointly normal, then for $s \in \{h, l\}$,

$$EU = -\pi(e^{-\gamma\mu_W(h)+\frac{\gamma^2}{2}\sigma^2_W(h)}) - (1-\pi)(e^{-\gamma\mu_W(l)+\frac{\gamma^2}{2}\sigma^2_W(l)}),$$

where

$$\begin{aligned}\mu_W(s) =& E[\tilde{W}|\tilde{\rho} = \rho_s] = W_0 R + \rho_s\bar{\delta}(q_A + q_B) + \bar{V}(q_A + q_B) - R(q_A P_A + q_B P_B), \text{ and}\\ \sigma^2_W(s) =& V[\tilde{W}|\tilde{\rho} = \rho_s] = q_A^2(\sigma^2_V + \rho_s^2\sigma^2_\delta) + q_B^2(\sigma^2_V + \rho_s^2\sigma^2_\delta) + 2q_A q_B \rho_s^2\sigma^2_\delta + 2q_A^2\rho_s\sigma_{\delta\iota} + 2q_B^2\rho_s\sigma_{\delta\iota}.\end{aligned}$$

Take the partial derivative of $EU$ with respect to $q_i$, then setting $q_i = 1/2$ results in the price for firm $i$ being

$$\begin{aligned}P_i = & \frac{\bar{V} - \frac{\gamma}{2}\sigma^2_V}{R} + \frac{\bar{\delta} - \gamma\sigma_{\delta\iota}}{R}\left[\frac{\pi\rho_h e^{-\gamma\bar{\delta}\rho_h + \frac{\gamma^2}{2}(\rho_h^2\sigma_\delta^2 + \rho_h\sigma_{\delta\iota})} + (1-\pi)\rho_l e^{-\gamma\bar{\delta}\rho_l + \frac{\gamma^2}{2}(\rho_l^2\sigma_\delta^2 + \rho_l\sigma_{\delta\iota})}}{\pi e^{-\gamma\bar{\delta}\rho_h + \frac{\gamma^2}{2}(\rho_h^2\sigma_\delta^2 + \rho_h\sigma_{\delta\iota})} + (1-\pi) e^{-\gamma\bar{\delta}\rho_l + \frac{\gamma^2}{2}(\rho_l^2\sigma_\delta^2 + \rho_l\sigma_{\delta\iota})}}\right]\\ & - \frac{\gamma\sigma^2_\delta}{R}\left[\frac{\pi\rho_h^2 e^{-\gamma\bar{\delta}\rho_h + \frac{\gamma^2}{2}(\rho_h^2\sigma_\delta^2 + \rho_h\sigma_{\delta\iota})} + (1-\pi)\rho_l^2 e^{-\gamma\bar{\delta}\rho_l + \frac{\gamma^2}{2}(\rho_l^2\sigma_\delta^2 + \rho_l\sigma_{\delta\iota})}}{\pi e^{-\gamma\bar{\delta}\rho_h + \frac{\gamma^2}{2}(\rho_h^2\sigma_\delta^2 + \rho_h\sigma_{\delta\iota})} + (1-\pi) e^{-\gamma\bar{\delta}\rho_l + \frac{\gamma^2}{2}(\rho_l^2\sigma_\delta^2 + \rho_l\sigma_{\delta\iota})}}\right].\end{aligned}$$

The FOC of the expected asset price with the described correlation structure with respect to $\alpha$ equals zero at $\alpha = \frac{1}{2}$ is:

$$\left.\frac{\partial E[P_j(\alpha;\tilde{m})]}{\partial\alpha}\right|_{\alpha=\frac{1}{2}} = 0.$$

The relevant expression for the sign of the second-order condition (SOC) $\frac{\partial^2 E[P_j(\alpha;\tilde{m})]}{\partial\alpha^2}$ for $\alpha = \frac{1}{2}$ to be a maximum is given by

$$\left[\gamma\sigma^2_\delta(\rho_h + \rho_l) + \gamma\sigma_{\delta\iota} - \bar{\delta}\right]\left(e^{\frac{1}{2}\gamma^2(\rho_h^2\sigma_\delta^2 + \rho_h\sigma_{\delta\iota}) - \gamma\rho_h\bar{\delta}} - e^{\frac{1}{2}\gamma^2(\rho_l^2\sigma_\delta^2 + \rho_l\sigma_{\delta\iota}) - \gamma\rho_l\bar{\delta}}\right) < 0;$$

otherwise $\alpha = \frac{1}{2}$ is a minimum.

Using the same method from 2, the only feasible conditions for the SOC to be satisfied is:

$$\tfrac{1}{2}(\gamma\sigma^2_\delta(\rho_h + \rho_l) + \gamma\sigma_{\delta\iota}) < \bar{\delta} < \gamma\sigma^2_\delta(\rho_h + \rho_l) + \gamma\sigma_{\delta\iota}.$$

## Proof of Proposition 7

The first statement of the proof is straightforward as expected utility when there is a relationship is the same under disclosure and under non-disclosure. The reason is that the final wealth in equilibrium is given by $\tilde{W} = \frac{1}{2}(\tilde{F}_A + \tilde{F}_B)$. Denote $EU_R$ as the expected utility when there is a relationship (either with disclosure or non-disclosure), which is given by

$$EU_R = E[-e^{-\gamma \tilde{W}}] = -e^{-\gamma \bar{V} + \frac{\gamma^2}{4}\sigma_V^2} \left[ \pi e^{-\gamma \rho_h \bar{\delta} + \frac{\gamma^2}{2}\rho_h^2 \sigma_\delta^2} + (1-\pi) e^{-\gamma \rho_l \bar{\delta} + \frac{\gamma^2}{2}\rho_l^2 \sigma_\delta^2} \right].$$

Denote $EU_{NR}$ as the expected utility when there is no relationship, which is given by

$$EU_{NR} = E[-e^{-\gamma \tilde{W}}] = -e^{-\gamma \bar{V} + \frac{\gamma^2}{4}\sigma_V^2}.$$

$EU_R > EU_{NR}$ if and only if

$$1 > \left[ \pi e^{-\gamma \rho_h \bar{\delta} + \frac{\gamma^2}{2}\rho_h^2 \sigma_\delta^2} + (1-\pi) e^{-\gamma \rho_l \bar{\delta} + \frac{\gamma^2}{2}\rho_l^2 \sigma_\delta^2} \right].$$

A sufficient condition for $EU_R > EU_{NR}$ is given by

$$\frac{\rho_h + \rho_l}{2} < \frac{\bar{\delta}}{\gamma \sigma_\delta^2}.$$

Hence, if the condition in Proposition 4 is satisfied

$$\max \left\{ \frac{\rho_h + \rho_l}{2}, \frac{\pi \rho_h^2 e^{\frac{1}{2}\gamma^2 \rho_h^2 \sigma_\delta^2 - \gamma \rho_h \bar{\delta}} + (1-\pi)\rho_l^2 e^{\frac{1}{2}\gamma^2 \rho_l^2 \sigma_\delta^2 - \gamma \rho_l \bar{\delta}}}{\pi \rho_h e^{\frac{1}{2}\gamma^2 \rho_h^2 \sigma_\delta^2 - \gamma \rho_h \bar{\delta}} + (1-\pi)\rho_l e^{\frac{1}{2}\gamma^2 \rho_l^2 \sigma_\delta^2 - \gamma \rho_l \bar{\delta}}} \right\} < \frac{\bar{\delta}}{\gamma \sigma_\delta^2} < \frac{\pi \rho_h^2 + (1-\pi)\rho_l^2}{\pi \rho_h + (1-\pi)\rho_l},$$

then mandatory disclosure destroys relationships and decreases the welfare of investors.

# Online Appendix

## OA1 Technical details of introducing correlation between firm-specific cash flow and cash flow from relationship formation

Denote Cov($\tilde{V}_i, \tilde{\delta}$) = $\sigma_{\delta i}$. We still assume that $\tilde{V}_1$and $\tilde{\delta}$ are jointly normal, then for $s \in \{h, l\}$,

$$EU = -\pi(e^{-\gamma\mu_W(h)+\frac{\gamma^2}{2}\sigma^2_W(h)}) - (1-\pi)(e^{-\gamma\mu_W(l)+\frac{\gamma^2}{2}\sigma^2_W(l)}),$$

where

$$\mu_W(s) = E[\tilde{W}|\tilde{\rho} = \rho_s] = W_0R + \rho_s\bar{\delta}(q_A + q_B) + \bar{V}(q_A + q_B) - R((q_AP_A + q_BP_B)),$$
$$\sigma^2_W(s) = V[\tilde{W}|\tilde{\rho} = \rho_s] = q_A^2(\sigma_V^2 + \rho_s^2\sigma_\delta^2) + q_B^2(\sigma_V^2 + \rho_s^2\sigma_\delta^2) + 2q_Aq_B\rho_s^2\sigma_\delta^2 + 2q_A^2\rho_s\sigma_{\delta\iota} + 2q_B^2\rho_s\sigma_{\delta\iota}.$$

Take the partial derivative of $EU$ with respect to $q_i$, then setting $q_i = 1/2$ results in the price for firm $i$ being

$$P_i = \frac{\bar{V} - \frac{\gamma}{2}\sigma_V^2}{R} + \frac{\bar{\delta} - \gamma\sigma_{\delta\iota}}{R}\left[\frac{\pi\rho_h e^{-\gamma\bar{\delta}\rho_h+\frac{\gamma^2}{2}(\rho_h^2\sigma_\delta^2+\rho_h\sigma_{\delta\iota})} + (1-\pi)\rho_l e^{-\gamma\bar{\delta}\rho_l+\frac{\gamma^2}{2}(\rho_l^2\sigma_\delta^2+\rho_l\sigma_{\delta\iota})}}{\pi e^{-\gamma\bar{\delta}\rho_h+\frac{\gamma^2}{2}(\rho_h^2\sigma_\delta^2+\rho_h\sigma_{\delta\iota})} + (1-\pi)e^{-\gamma\bar{\delta}\rho_l+\frac{\gamma^2}{2}(\rho_l^2\sigma_\delta^2+\rho_l\sigma_{\delta\iota})}}\right]$$
$$- \frac{\gamma\sigma_\delta^2}{R}\left[\frac{\pi\rho_h^2 e^{-\gamma\bar{\delta}\rho_h+\frac{\gamma^2}{2}(\rho_h^2\sigma_\delta^2+\rho_h\sigma_{\delta\iota})} + (1-\pi)\rho_l^2 e^{-\gamma\bar{\delta}\rho_l+\frac{\gamma^2}{2}(\rho_l^2\sigma_\delta^2+\rho_l\sigma_{\delta\iota})}}{\pi e^{-\gamma\bar{\delta}\rho_h+\frac{\gamma^2}{2}(\rho_h^2\sigma_\delta^2+\rho_h\sigma_{\delta\iota})} + (1-\pi)e^{-\gamma\bar{\delta}\rho_l+\frac{\gamma^2}{2}(\rho_l^2\sigma_\delta^2+\rho_l\sigma_{\delta\iota})}}\right].$$

### OA1.1 Relationship Disclosure

After sending message $\tilde{m}$, the price of firm $i$ with the posterior probability $\pi^m$ can be computed in the same manner as the original model. Each firm $j$ would thus choose disclosure policy $\alpha$ to maximize the expected price

$$\mathbb{E}[P_i(\alpha; \tilde{m})] = P_i(\alpha; \tilde{m} = h)Pr(\tilde{m} = h) + P_i(\alpha; \tilde{m} = l)Pr(\tilde{m} = l).$$

### OA1.2 Relationship formation

Mandatory disclosure will destroy relationships if and only if the following two conditions are satisfied: 1) for any firm $j \in \{A, B\}$ we have that $E[P_j(\alpha = \frac{1}{2}; \tilde{m})] > P_j^{NR} > E[P_j(\alpha = 1; \tilde{m})]$; and 2) it is optimal for firms not to disclose, hence $\alpha^* = \frac{1}{2}$ is optimal following Proposition 6.

These two conditions are satisfied if and only if

$$\frac{(\pi\rho_h^2+(1-\pi)\rho_l^2)}{(\pi\rho_h+(1-\pi)\rho_l)} > \frac{\left(\bar{\delta}-\gamma\sigma_\delta\right)}{\gamma\sigma_\delta^2}$$

$$> max\left\{\frac{1}{2}[(\rho_h+\rho_l)-\frac{\sigma_{\delta\iota}}{\sigma_\delta}], \frac{\pi\rho_h^2 e^{-\gamma\bar{\delta}\rho_h+\frac{\gamma^2}{2}(\rho_h^2\sigma_\delta^2+\rho_h\sigma_{\delta\iota})}+(1-\pi)\rho_l^2 e^{-\gamma\bar{\delta}\rho_l+\frac{\gamma^2}{2}(\rho_l^2\sigma_\delta^2+\rho_l\sigma_{\delta\iota})}}{\pi\rho_h e^{-\gamma\bar{\delta}\rho_h+\frac{\gamma^2}{2}(\rho_h^2\sigma_\delta^2+\rho_h\sigma_{\delta\iota})}+(1-\pi)\rho_l e^{-\gamma\bar{\delta}\rho_l+\frac{\gamma^2}{2}\left(\rho_l^2\sigma_\delta^2+\rho_l\sigma_{\delta\iota}\right)}}\right\}.$$

# OA2 Other variations

## OA2.1 Optimal collaboration intensity

In this subsection, we study the optimal collaboration intensity $\bar{\delta}$ from a firm's point of view. We continue to assume that each firm's objective function is to maximize the expected asset price. Since the two firms are symmetric, we focus on one firm without loss of generality. To illustrate the result most transparently, we assume that it is costless for each firm to change $\bar{\delta}$, which is a measure for the intensive margin of firm relationships.

For the case without matching uncertainty, by pricing function (4), firms would choose the largest $\bar{\delta}$ possible (i.e., with unbounded support, firms would choose $\bar{\delta}$ to be infinity).

We now consider the case with matching uncertainty and in this case, we show that the optimal intensity $\bar{\delta}$ is interior. For simplicity, we assume that $\rho_l = 0$. As mentioned before, in this special setting, uncertainty about matching intensity $\tilde{\rho}$ can also be interpreted as uncertainty about the existence of firm relationship. The asset prices for $j = \{A, B\}$ are just a special case of the prices in the conceptual framework given by equation (6):

$$\begin{aligned} P_j &= \frac{\bar{V}-\frac{\gamma}{2}\sigma_V^2}{R} \\ &+ \left(\frac{\bar{\delta}}{R}-\frac{\gamma\sigma_\delta^2\rho_h}{R}\right)\left[\frac{\pi\rho_h e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2-\gamma\rho_h\bar{\delta}}}{\pi e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2-\gamma\rho_h\bar{\delta}}+(1-\pi)}\right]. \end{aligned}$$

Firms choose their collaboration intensity $\bar{\delta} \in [0, \infty)$ to maximize their asset prices (OA1).

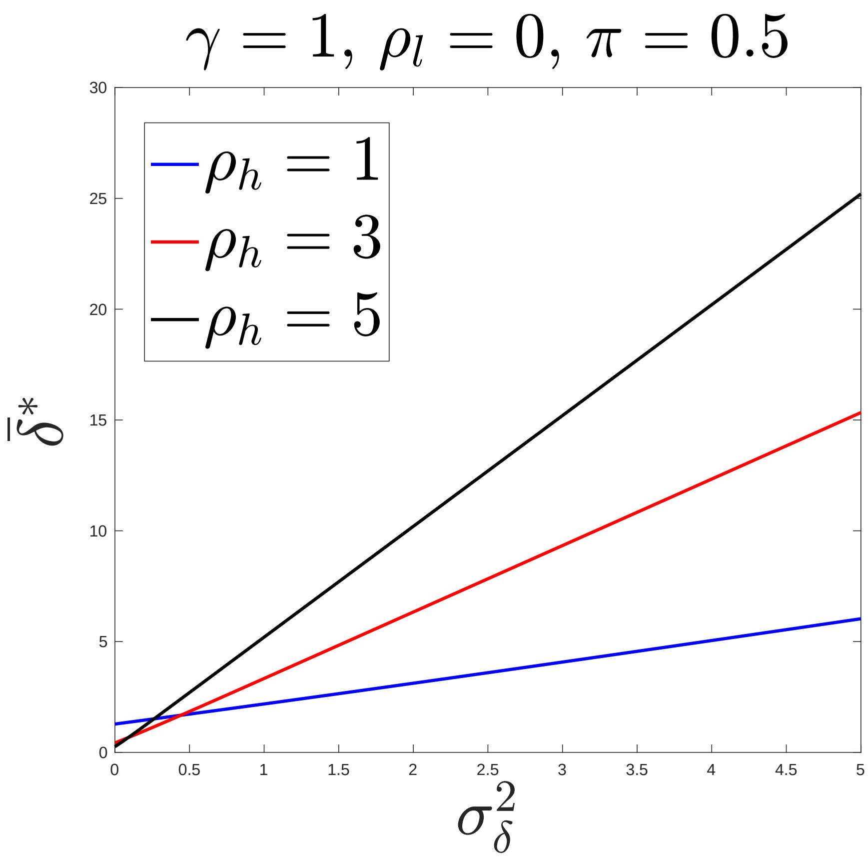
An increase in $\bar{\delta}$ has two effects on the price (OA1). There is a direct effect that increases the price due to an increase in expected cash flows. There is also an indirect effect that decreases the price due to the risk associated with an increase in the asset demand of both assets caused by the direct effect. A high asset demand adds risk to the investor's portfolio as she believes with probability $1-\pi$ that firms $A$ and $B$ have no relationship ($\tilde{\rho} = \rho_l = 0$). In other words, an increase in $\bar{\delta}$ increases the downside risk and thus the kurtosis term as captured in the third term of equation (OA1). When the investor is uncertain about the matching intensity between the two firms, it is optimal for firms to have a limited collaboration intensity, in contrast to the case in which the investor is certain about the matching intensity.

**Proposition OA1** (Optimal Collaboration Intensity)**.** *The optimal collaboration intensity* $\bar{\delta} = \bar{\delta}^*$ *is uniquely determined by the solution to the following equation:*

$$\pi(1-\pi)\rho_h + \rho_h \pi^2 e^{\frac{1}{2}\gamma^2 \rho_h^2 \sigma_\delta^2 - \gamma \rho_h \bar{\delta}} + \gamma^2 \pi(1-\pi)\rho_h^3 \sigma_\delta^2 - \gamma \pi (1-\pi) \rho_h^2 \bar{\delta} = 0.$$

In Figure OA1, we report the optimal collaboration intensity $\bar{\delta}^*$ for several parameter specifications. Specifically, we assume $\gamma = 1$ and $\pi = 0.5$, set several values for $\rho_h$, and depict $\bar{\delta}^*$ against $\sigma_\delta^2$. We can see that the optimal collaboration intensity $\bar{\delta}^*$ is interior and increasing in $\sigma_\delta^2$.

Figure OA1: Optimal Collaboration Intensity



### Proof of Proposition OA1

Taking the FOC of the asset price under uncertainty (6) with respect to $\bar{\delta}$ and re-arranging terms, we obtain

$$e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2-\gamma\rho_h\bar{\delta}}\left[\frac{\pi(1-\pi)\rho_h+\rho_h\pi^2 e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2-\gamma\rho_h\bar{\delta}}+\gamma^2\pi(1-\pi)\rho_h^3\sigma_\delta^2-\gamma\pi(1-\pi)\rho_h^2\bar{\delta}}{R\left(\pi e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2-\gamma\rho_h\bar{\delta}}+(1-\pi)\right)^2}\right]=0.$$

## OA2.2 Disclosure of exposure to factors by one firm

This subsection aims at further comparing our paper to Heinle et al. (2018) by connecting the disclosure of an exposure to a risk factor as in their study to a disclosure of a relationship intensity as in our paper. Assume there is only one firm with payoffs $\tilde{F}_A=\tilde{V}_A+\tilde{\Delta}$, where $\tilde{V}_A\sim N(\bar{V},\sigma_V^2)$ and $\tilde{\Delta}$ is given by

$$\tilde{\Delta}=\tilde{\rho}\tilde{\delta},\text{ with }\tilde{\delta}\sim N\left(\bar{\delta},\sigma_\delta^2\right)\text{ and }\tilde{\rho}=\begin{cases}\rho_h & \text{with probability }\pi,\\ \rho_l & \text{with probability }1-\pi,\end{cases}\tag{OA1}$$

where $\rho_h>\rho_l\geq 0$ and $\bar{\delta}\geq 0$. Investors are uncertain about the exposure of the firm to the factor $\tilde{\delta}$. The price of the asset is given by

$$\begin{aligned}P_A &= \frac{\bar{V}-\frac{\gamma}{2}\sigma_V^2}{R}\\ &+ \frac{\bar{\delta}}{R}\left[\frac{\pi\rho_h e^{\frac{1}{8}\gamma^2\rho_h^2\sigma_\delta^2-\frac{1}{2}\gamma\rho_h\bar{\delta}}+(1-\pi)\rho_l e^{\frac{1}{8}\gamma^2\rho_l^2\sigma_\delta^2-\frac{1}{2}\gamma\rho_l\bar{\delta}}}{\pi e^{\frac{1}{8}\gamma^2\rho_h^2\sigma_\delta^2-\frac{1}{2}\gamma\rho_h\bar{\delta}}+(1-\pi)e^{\frac{1}{8}\gamma^2\rho_l^2\sigma_\delta^2-\frac{1}{2}\gamma\rho_l\bar{\delta}}}\right]\\ &- \frac{\gamma\sigma_\delta^2}{2R}\left[\frac{\pi\rho_h^2 e^{\frac{1}{8}\gamma^2\rho_h^2\sigma_\delta^2-\frac{1}{2}\gamma\rho_h\bar{\delta}}+(1-\pi)\rho_l^2 e^{\frac{1}{8}\gamma^2\rho_l^2\sigma_\delta^2-\frac{1}{2}\gamma\rho_l\bar{\delta}}}{\pi e^{\frac{1}{8}\gamma^2\rho_h^2\sigma_\delta^2-\frac{1}{2}\gamma\rho_h\bar{\delta}}+(1-\pi)e^{\frac{1}{8}\gamma^2\rho_l^2\sigma_\delta^2-\frac{1}{2}\gamma\rho_l\bar{\delta}}}\right].\end{aligned}\tag{OA2}$$

There are three changes with respect to the price under matching uncertainty with two firms given by equation (6). First, the third term is now divided by 2, which accounts for the diversification cost, as there is no diversification cost with a single firm. Second, there is a change in the relative importance of the variance of the common factor in the exponential weights relative to the mean of this common factor. The variance of the common factor in this one-firm case is relatively less important than the mean relative to the two-firm case. Third, there is a change in the absolute importance of weights. The exponential weights are smaller than in the two-firm case due to the absence of correlation in the one-firm case.

As in Section 4, we now specify that the firm can commit to a disclosure policy such that the firm can choose the following probability $\alpha \in [\frac{1}{2}, 1]$ at zero cost:

$$\begin{aligned} Pr(\tilde{m} = h \mid \tilde{\rho} = \rho_h) &= Pr(\tilde{m} = l \mid \tilde{\rho} = \rho_l) = \alpha, \\ Pr(\tilde{m} = l \mid \tilde{\rho} = \rho_h) &= Pr(\tilde{m} = h \mid \tilde{\rho} = \rho_l) = 1 - \alpha. \end{aligned} \tag{OA3}$$

As before, when $\alpha = 1$, the firm provides perfect disclosure of the realization of $\tilde{\rho}$. Instead, when $\alpha = 1/2$, the firm provides no information about the realization of $\tilde{\rho}$. Any $\alpha$ in between will generate only partial disclosure about the realization of $\tilde{\rho}$. The following proposition shows the conditions under which the firm will choose to opt for a full disclosure policy or a non-disclosure policy, which is a counterpart for Proposition 2. The intuition is similar to the model in our main setting but the results are qualitatively different. Specifically, the region where no disclosure is optimal shrinks. This is because no disclosure is more costly due to the absence of diversification cost. In addition, since there is only one firm, we are not able to answer questions related to the real effects of mandatory relationship disclosure on relationship formation.

**Proposition OA2.** *The optimal disclosure policy $\alpha = \alpha^*$ is a corner solution and is given by*

$$\alpha^* = \begin{cases} \frac{1}{2}, & \text{if } \frac{\rho_l + \rho_h}{4} < \frac{\bar{\delta}}{\gamma \sigma_\delta^2} < \frac{\rho_l + \rho_h}{2}, \\ 1, & \text{otherwise}. \end{cases} \tag{OA4}$$

## Proof of Proposition OA2

Following the same steps as in the proof of Proposition 2, the FOC of the expected asset price (OA2) with respect to $\alpha$ equals zero at $\alpha = \frac{1}{2}$ is:

$$\left. \frac{\partial E[P_A(\alpha; \tilde{m})]}{\partial \alpha} \right|_{\alpha = \frac{1}{2}} = 0.$$

The disclosure policy $\alpha = \frac{1}{2}$ is a maximium if and only if these two conditions are satisfied: $2\bar{\delta} < \gamma(\rho_l + \rho_h)\sigma_\delta^2$ and $e^{\frac{1}{8}\gamma^2\rho_h^2\sigma_\delta^2 - \frac{1}{2}\gamma\rho_h\bar{\delta}} < e^{\frac{1}{8}\gamma^2\rho_l^2\sigma_\delta^2 - \frac{1}{2}\gamma\rho_l\bar{\delta}}$.